1. RELATIONS & FUNCTIONS

March, 2025

MULTIPLE CHOICE QUESTIONS (1 Mark)

1. For real x, let $f(x) = x^3 + 5x + 1$. Then :
(A) f is one-one but not onto on $\mathbb{R}$
(B) f is onto on $\mathbb{R}$ but not one-one
(C) f is one-one and onto on $\mathbb{R}$
(D) f is neither one-one nor onto on $\mathbb{R}$

2. If $f : \mathbb{N} \to \mathbb{W}$ is defined as

$$f(n) = \begin{cases} \frac{n}{2} \text{ , if } n \text{ is even} \\ 0 \text{ , if } n \text{ is odd} \end{cases}$$

then f is :
(A) injective only
(B) surjective only
(C) a bijection
(D) neither surjective nor injective

3. Which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is both one-one and onto ?
(A) $f(x) = 2x - 1$
(B) $f(x) = 3x^2 + 5$
(C) $f(x) = x + 5$
(D) $f(x) = 5x^3$

4. Let A = {a, b}, then the number of reflexive relations defined on A is :
(A) 16
(B) 8
(C) 4
(D) 2

ASSERTION - REASON QUESTIONS (1 Mark)

Directions: In the following questions (5-7), a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices:
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

5. **Assertion (A)** : Let $\mathbb{Z}$ be the set of integers. A function $f : \mathbb{Z} \to \mathbb{Z}$ defined as $f(x) = 3x - 5, \ \forall x \in \mathbb{Z}$ is a bijective.
Reason (R) : A function is a bijective if it is both surjective and injective.

6. **Assertion (A)** : Let A $= \{x \in \mathbb{R} : -1 \leq x \leq 1\}$. If $f : A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.
Reason (R) : If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

7. Consider the function $f : \mathbb{R} \to \mathbb{R}$, defined as $f(x) = x^3$.
Assertion (A) : $f(x)$ is a one-one function.
Reason (R) : $f(x)$ is a one-one function, if co-domain = range.

VERY SHORT ANSWER QUESTIONS (2 Marks)

8. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$.
Discuss the bijectivity of the function.

SHORT ANSWER & LONG ANSWER QUESTIONS (3 - 5 Marks)

9. Let R be a relation defined over $\mathbb{N}$, where $\mathbb{N}$ is set of natural numbers, defined as "mRn if and only if m is a multiple of n, m, n $\in \mathbb{N}$." Find whether R is reflexive, symmetric and transitive or not.

10. (a) If $f : \mathbb{R}^+ \to \mathbb{R}$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection. ($\mathbb{R}^+$ is a set of all positive real numbers.)

OR

(b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as R = {(x, y) : $x + y = 6, x \in A, y \in B$}.

(i) Write all elements of R.
(ii) Is R a function ? Justify.
(iii) Determine domain and range of R.

11. (a) Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 4x^3 - 5,\ \forall\ x \in \mathbb{R}$ is one-one and onto.

OR

(b) Let R be a relation defined on a set $\mathbb{N}$ of natural numbers such that $R = \{(x, y) :$ xy is a square of a natural number, $x, y \in \mathbb{N}\}$. Determine if the relation R is an equivalence relation.

12. Let A be the set of all positive integers and a relation R on $A \times A$ is defined by $(a, b)R(c, d) \Leftrightarrow ad = bc$, for all $(a, b), (c, d) \in A \times A$. Show that R is an equivalence relation on $A \times A$.

CASE STUDY BASED QUESTIONS (4 Marks)

13. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set A = {1, 2, 3} :
$R_1 = \{(2, 3), (3, 2)\}$
$R_2 = \{(1, 2), (1, 3), (3, 2)\}$
$R_3 = \{(1, 2), (2, 1), (1, 1)\}$
$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$
$R_5 =$
$\{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$
The students are asked to answer the following questions about the above relations :
(i) Identify the relation which is reflexive, transitive but not symmetric.
(ii) Identify the relation which is reflexive and symmetric but not transitive.
(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation ?

14. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) :$ speaker x is judged by judge $y,\ x \in S,\ y \in J\}$.
Based on the above, answer the following:
(i) How many relations can be there from S to J? **1**
(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$. Check if it is bijective. **1**
(iii) (a) How many one-one functions can be there from set S to set J? **2**

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), (S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**

15. Let A be the set of 30 students of class XII in a school. Let f : A $\to$ $\mathbb{N}$, $\mathbb{N}$ is a set of natural numbers such that function f(x) = Roll Number of student x.
On the basis of the given information, answer the following :
(i) Is f a bijective function ?
(ii) Give reasons to support your answer to (i).
(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where
$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = 3x\}$.
List the elements of R. Is the relation R reflexive, symmetric and transitive ? Justify your answer.

OR

(iii) (b) Let R be a relation defined by
$R = \{(x, y) : x, y$ are Roll Numbers of students such that $y = x^3\}$.
List the elements of R. Is R a function ? Justify your answer.

16. During the festival season, there was a mela organized by the Resident Welfare Association at a park, near the society. The main attraction of the mela was a huge swing installed at one corner of the park. The swing is traced to follow the path of a parabola given by $x^2 = y$.

Based on the above information, answer the following questions :

(i) Let $f : \mathbb{N} \to \mathbb{R}$ is defined by $f(x) = x^2$. What will be the range ? **1**

(ii) Let $f : \mathbb{N} \to \mathbb{N}$ is defined by $f(x) = x^2$. Check if the function is injective or not. **1**

(iii) (a) Let $f : \{1, 2, 3, 4.....\} \to \{1, 4, 9, 16.....\}$ be defined by $f(x) = x^2$. Prove that the function is bijective. **2**

OR

(iii) (b) Let $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^2$. Show that f is neither injective nor surjective. **2**

17. Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions :

(i) Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined as $f(x) = x^2$. Find whether f is one-one function.

(ii) Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^2$. Find whether f is an onto function.

(iii) (a) Let $f : \mathbb{N} \to \mathbb{N}$ be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function.

OR

(iii) (b) Let $f : \mathbb{N} \to \{1, 4, 9, 16, ...\}$ defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.

March, 2024

SECTION A: Multiple Choice Questions (1 Mark)

1. A function f : $\mathbb{R}_+ \to \mathbb{R}$ (where $\mathbb{R}_+$ is the set of all non-negative real numbers) defined by f(x) = 4 x + 3 is :
(A) one-one but not onto
(B) onto but not one-one
(C) both one-one and onto
(D) neither one-one nor onto

2. Let f : $\mathbb{R}_+ \to [-5, \infty)$ be defined as $f(x) = 9x^2 + 6x - 5$, where $\mathbb{R}_+$ is the set of all non-negative real numbers. Then, f is :
(A) one-one
(B) onto
(C) bijective
(D) neither one-one nor onto

3. Let $\mathbb{R}_+$ denote the set of all non-negative real numbers. Then the function f : $\mathbb{R}_+ \to \mathbb{R}_+$ defined as $f(x) = x^2 + 1$ is :
(A) one-one but not onto
(B) onto but not one-one
(C) both one-one and onto
(D) neither one-one nor onto

4. (Assertion-Reason)
Assertion (A) : The relation $R = \{(x, y) : (x + y)$ is a prime number and $x, y \in \mathbb{N}\}$ is not a reflexive relation.
Reason (R) : The number '2n' is composite for all natural numbers n.
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

5. A function f : $\mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2 - 4x + 5$ is :
(A) injective but not surjective.
(B) surjective but not injective.
(C) both injective and surjective.
(D) neither injective nor surjective.

SECTION C: Short Answer Type Questions (3 Marks)

6. (a) A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $\mathbb{R} \to \mathbb{R}$ as $f(x) = ax + b$, such that f(1) = 1 and f(2) = 3. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

SECTION D: Long Answer Type Questions (5 Marks)

7. (a) Show that a function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : \mathbb{R} \to A$ becomes an onto function.

OR

(b) A relation R is defined on $\mathbb{N} \times \mathbb{N}$ (where N is the set of natural numbers) as :
(a, b) R (c, d) ⇔ a - c = b - d
Show that R is an equivalence relation.

8. A relation R on set A = {- 4, - 3, - 2, - 1, 0, 1, 2, 3, 4} be defined as R = {(x, y) : x + y is an integer divisible by 2}. Show that R is an equivalence relation. Also, write the equivalence class [2].

9. (a) Let $A = \mathbb{R} - \{5\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f : A \to B$, defined by $f(x) = \frac{x-3}{x-5}$. Show that f is one-one and onto.

OR

(b) Check whether the relation S in the set of real numbers $\mathbb{R}$ defined by $S = \{(a, b) :$ where $a - b + \sqrt{2}$ is an irrational number$\}$ is reflexive, symmetric or transitive.

10. (a) Let $A = \{x \mid x \in \mathbb{Z}, 0 \le x \le 12\}$. Show that the relation $R = \{(a, b) : a, b \in A, (a - b)$ is divisible by $4\}$ is an equivalence relation. Find the set of elements related to 2.

OR

(b) Let $A = \mathbb{R} - \{4\}$ and $B = \mathbb{R} - \{1\}$ and let function $f : A \to B$ be defined as $f(x) = \frac{x-3}{x-4}$ for $\forall x \in A$. Show that f is one-one and onto.

11. Check whether the relation S in the set of all real numbers ($\mathbb{R}$) defined by
$S = \{(a, b) : a \le b^3\}$ is reflexive, symmetric or transitive.

SECTION E: Case Study Based Questions (4 Marks)

12. (a) Students of a school are taken to a railway museum to learn about railways heritage and its history.
An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by
$R = \{(l_1, l_2) : l_1$ is parallel to $l_2\}$
On the basis of the above information, answer the following questions :
(i) Find whether the relation R is symmetric or not.
(ii) Find whether the relation R is transitive or not.
(iii) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it.

OR

(b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1$ is perpendicular to $l_2\}$ check whether the relation S is symmetric and transitive.

March, 2023

SECTION A

1. Let A = {3, 5}. Then number of reflexive relations on A is
(a) 2
(b) 4
(c) 0
(d) 8

SECTION B

2. A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B.

3. Check the injectivity and surjectivity of the function $f : \mathbb{N} \to \mathbb{N}$, given by $f(x) = x^3$.

4. Prove that the greatest integer function $f : \mathbb{R} \to \mathbb{R}$, given by $f(x) = [x]$, is neither one-one nor onto.

5. (a) Let the relation R be given as
$R = \{(x, y) : x, y \in \mathbb{N}$ and $x + 3y = 12\}$.
Find the domain and range of R.

OR

(b) Show that the function $f : \mathbb{R} \to \mathbb{R}, f(x) = x^4$ is neither one-one nor onto.

SECTION D

6. A relation R is defined on a set of real numbers $\mathbb{R}$ as
$R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$.
Check whether R is reflexive, symmetric and transitive or not.

7. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function. Further, find all possible values of 'a' for which $f(a) = \sqrt{7}$.

8. Show that a function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = \frac{5x-3}{4}$ is both one-one and onto.

9. (a) If N denotes the set of all natural numbers and R is the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)R(c, d)$, if $ad(b + c) = bc(a + d)$. Show that R is an equivalence relation.

OR

(b) Let $f : \mathbb{R} - \left\{-\frac{4}{3}\right\} \to \mathbb{R}$ be a function defined as $f(x) = \frac{4x}{3x+4}$. Show that f is a one-one function. Also, check whether f is an onto function or not.

10. (a) A relation R in the set A = {5, 6, 7, 8, 9} is given by R = {(x, y) : $|x - y|$ is divisible by 2}. Write R in roster form and prove that R is an equivalence relation. Also, find the elements related to element 7.

OR

(b) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$ be two sets. Prove that the function $f : A \to B$ given by $f(x) = \frac{x-2}{x-3}$ is onto. Is the function f one-one ? Justify your answer.

11. (a) Show that the relation S in set $\mathbb{R}$ of real numbers defined by
$S = \{(a, b) : a \leq b^3,\ a \in \mathbb{R},\ b \in \mathbb{R}\}$
is neither reflexive, nor symmetric, nor transitive.

OR

(b) Let R be the relation defined in the set $A = \{1, 2, 3, 4, 5, 6, 7\}$ by
$R = \{(a, b) :$ both a and b are either odd or even}. Show that R is an equivalence relation. Hence, find the elements of equivalence class [1].

12. (a) Let R be a relation in $\mathbb{R}$, the set of all real numbers, defined by $R = \{(a, b) : a \leq b^3\}$. Show that R is neither reflexive, nor symmetric and nor transitive.

OR

(b) Let set $A = \{1, 2, 3, ..., 10\}$ and R be a relation in $A \times A$, defined by $(a, b)R(c, d) \iff a + d = b + c$ for all (a, b) and $(c, d) \in A \times A$. Prove that R is an equivalence relation.

SECTION E

13. Case Study
An organization conducted bike race under two different categories – Boys and Girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.
[QP5 - Question 36]
Based on the above information, answer the following :
(I) How many relations are possible from B to G ?
(II) Among all the possible relations from B to G, how many functions can be formed from B to G ?
(III)
Let $R : B \to B$ be defined by $R = \{(x, y) : x \text{ and } y \text{ are students of the same sex}\}$.
Check if R is an equivalence relation.

OR

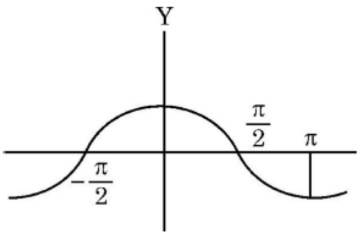
(III) A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$.
Check if f is bijective. Justify your answer.

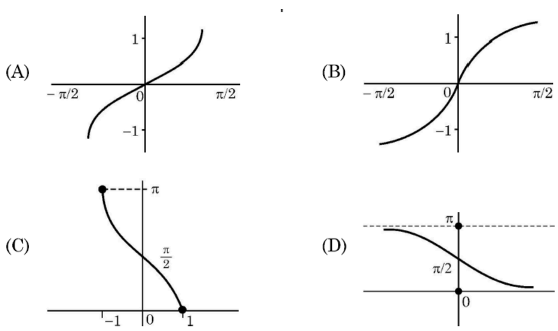
INVERSE TRIGONOMETRIC FUNCTIONS

MARCH, 2025

Multiple Choice Questions (1 Mark)

1. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse?



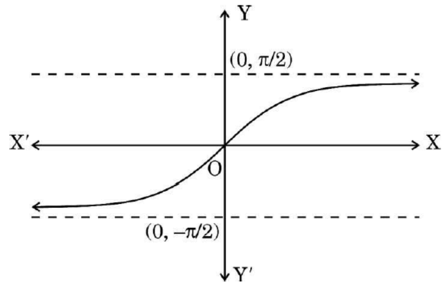


2. If $y = \sin^{-1} x, -1 \leq x \leq 0$, then the range of y is
(A) $[-\frac{\pi}{2}, 0)$
(B) $[-\frac{\pi}{2}, 0]$
(C) $[-\frac{\pi}{2}, 0)$
(D) $(-\frac{\pi}{2}, 0]$

3. The principal value of $\sin^{-1}\big(\sin\big(-\frac{10\pi}{3}\big)\big)$ is:
(A) $-\frac{2\pi}{3}$
(B) $-\frac{\pi}{3}$
(C) $\frac{\pi}{3}$
(D) $\frac{2\pi}{3}$

4. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is:
(A) $-\frac{\pi}{3}$
(B) $-\frac{2\pi}{3}$
(C) $\frac{\pi}{3}$
(D) $\frac{2\pi}{3}$

5. The given graph illustrates:



(A) $y = \tan^{-1} x$
(B) $y = \text{cosec}^{-1} x$
(C) $y = \cot^{-1} x$
(D) $y = \sec^{-1} x$

6. Domain of $f(x) = \cos^{-1} x + \sin x$ is:
(A) $\mathbb{R}$ (B) $(-1, 1)$
(C) $[-1, 1]$ (D) φ

7. The domain of $f(x) = \cos^{-1}(2x)$ is:
(A) $[-1, 1]$
(B) $\left[0, \frac{1}{2}\right]$
(C) $[-2, 2]$
(D) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

8. Value of $4\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is
(A) 3 (B) – 3 (C) 1 (D) – 1

Assertion - Reason Questions (1 Mark)

9. **Assertion (A)**: Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.
Reason (R): $\sec^{-1} x$ is defined for $x \in \mathbb{R} - (-1, 1)$.
(A) Both A and R are true and R is correct explanation of A.
(B) Both A and R are true, but R is **not** correct explanation.
(C) A is true, but R is false.
(D) A is false, but R is true.

Very Short Answer Questions (2 Marks)

10. Evaluate: $\tan^{-1}\left(2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right)$

11. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

OR

(b) Find domain of $\sin^{-1}\sqrt{x-1}$.

12. Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.

13. (a) Find principal value: $\cos^{-1}\left(-\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

OR

(b) Prove that: $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right), \quad x \in [0,1]$

14. Evaluate: $\tan^{-1}\left(\sqrt{3}\right) - \sec^{-1}(-2)$

15. Evaluate: $\sec^2(\tan^{-1} 3) + \text{cosec}^2(\cot^{-1} 2)$

March, 2024

Section A: Multiple Choice (1 Mark)

Directions for Q1-Q3: Assertion (A) is followed by Reason (R).
(A) Both A and R are true and R is correct explanation of A.
(B) Both A and R are true and R is not correct explanation.
(C) A is true, but R is false.
(D) A is false, but R is true.

1. Assertion (A): Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$.
Reason (R): The range of the principal value branch of $y = \cos^{-1}(x)$ is $[0, \pi] - \{\frac{\pi}{2}\}$.

2. Assertion (A): $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = \frac{\pi}{6}$
Reason (R): $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

3. Assertion (A): $\cos^{-1}\left(\cos\left(\frac{13\pi}{6}\right)\right)$ is equal to $\frac{\pi}{6}$.
Reason (R): The range of the principal value branch of the function $y = \cos^{-1} x$ is $[0, \pi]$.

Section B: Very Short Answer (2 Marks)

4. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}[\sin(-\frac{\pi}{2})]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

5. Evaluate: $\sec^2\left(\tan^{-1}\frac{1}{2}\right) + \text{cosec}^2\left(\cot^{-1}\frac{1}{3}\right)$

6. (a) Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$, where $\frac{-\pi}{2} < x < \frac{\pi}{2}$ in the simplest form.

OR

(b) Find the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$.

7. Find value of k if $\sin^{-1}\left[k\tan\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right] = \frac{\pi}{3}$.

8. (a) Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

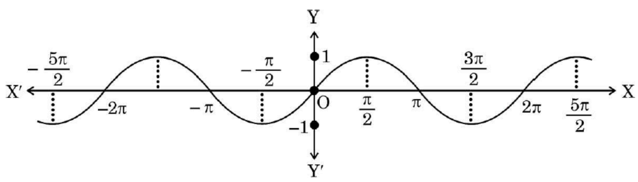
OR

(b) Find the domain of the function $y = \cos^{-1}(x^2 - 4)$.

9. Find value of $\cos^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \text{cosec}^{-1}(-2)$.

Section E: Case Study (4 Marks)

10. If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f. The domain of sine function is $\mathbb{R}$ and function sine : $\mathbb{R} \to \mathbb{R}$ is neither one-one nor onto. The following graph shows the sine function.



y = sin x

Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.
(i) If A is the interval other than principal value branch, give an example of one such interval.
(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$.
(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch.

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$.

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Section A

1. $\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

2. The domain of the function $\cos^{-1} x$ is:

(a) $[0, \pi]$ (b) $(-1, 1)$

(c) $[-1, 1]$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3. The domain of the function $\sin^{-1}(2x)$ is:

(a) $[-1, 1]$

(b) $[0, 1]$

(c) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(d) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Directions for Q4-Q8: Assertion (A) is followed by Reason (R).

(a) Both A and R are true and R is correct explanation of A.

(b) Both A and R are true, but R is **not** correct explanation.

(c) A is true and R is false.

(d) A is false and R is true.

4. Assertion (A): The range of the function $f(x) = 2\sin^{-1} x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$.

Reason (R): The range of the principal value branch of $\sin^{-1}(x)$ is $[0, \pi]$.

5. Assertion (A): Maximum value of $\left(\cos^{-1} x\right)^2$ is π^2.

Reason (R): Range of the principal value branch of $\cos^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

6. Assertion (A): Range of $[\sin^{-1} x + 2\cos^{-1} x]$ is $[0, \pi]$.

Reason (R): Principal value branch of $\sin^{-1} x$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

7. Assertion (A): All trigonometric functions have their inverses over their respective domains.

Reason (R): The inverse of $\tan^{-1} x$ exists for some $x \in \mathbb{R}$.

8. Assertion (A): The principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

Reason (R): Domain of $\cot^{-1} x$ is $\mathbb{R} - \{-1, 1\}$.

Section B

9. Evaluate: $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) + \cos^{-1}\left(\cos\left(\frac{3\pi}{4}\right)\right) + \tan^{-1}(1)$

10. (a) Evaluate $\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) + \cos^{-1}(\cos\pi) + \tan^{-1}(1)$.

OR

(b) Draw the graph of $\cos^{-1} x$, where $x \in [-1, 0]$. Also, write its range.

11. (a) Evaluate : $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}(0)$

OR

(b) Draw the graph of $f(x) = \sin^{-1} x, x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$. Also, write range of $f(x)$.

12. (a) Find the domain of $y = \sin^{-1}(x^2 - 4)$.

OR

(b) Evaluate: $\cos^{-1}\left[\cos\left(-\frac{7\pi}{3}\right)\right]$

13. Write the domain and range (principle value branch) of the following functions: $f(x) = \tan^{-1} x$

14. Find the value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.

15. Simplify: $\tan^{-1}\left(\frac{\cos x}{1 - \sin x}\right)$

3 & 4. MATRICES & DETERMINANTS

March, 2025

Multiple Choice Questions (1 Mark)

1. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. If A is a square matrix of order 2 such that $\det(A) = 4$, then $\det(4 \text{ adj } A)$ is equal to :

(A) 16 (B) 64
(C) 256 (D) 512

3. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$, $C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$, which of the following is defined ?

(A) Only AB
(B) Only AC
(C) Only BA
(D) All AB, AC and BA

4. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to

(A) 0 (B) 1
(C) 7 (D) ± 7

5. If A and B are invertible matrices, then which of the following is **not** correct ?

(A) $(A+B)^{-1} = B^{-1} + A^{-1}$
(B) $(AB)^{-1} = B^{-1}A^{-1}$
(C) adj $(A) = |A| \ A^{-1}$
(D) $|A|^{-1} = |A^{-1}|$

6. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is

(A) -8 (B) 0
(C) 6 (D) 8

7. Which of the following can be both a symmetric and skew-symmetric matrix?

(A) Unit Matrix
(B) Diagonal Matrix
(C) Null Matrix
(D) Row Matrix

8. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$. Their answers are given as:

Abhay : $6AB$
Bina : $7AB - BA$
Chhaya : $8AB$
Devesh : $7BA - AB$

Who answered it correctly?

(A) Abhay (B) Bina
(C) Chhaya (D) Devesh

9. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct?

(A) $A = B$
(B) $AB = BA$
(C) $A = 0$ or $B = 0$
(D) $A = I$ or $B = I$

10. If A and B are square matrices of same order such that $AB = A$ and $BA = B$, then $A^2 + B^2$ is equal to :

(A) $A + B$
(B) BA
(C) $2(A + B)$
(D) $2BA$

11. If M and N are square matrices of order 3 such that $\det(M) = m$ and $MN = mI$, then $\det(N)$ is equal to :

(A) -1 (B) 1
(C) $-m^2$ (D) m^2

12. The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a :

(A) diagonal matrix
(B) symmetric matrix
(C) skew symmetric matrix
(D) scalar matrix

13. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

(A) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(C) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$

(D) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

14. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is :

(A) Only AB is defined.
(B) Only BA is defined.
(C) AB and BA, both are defined.
(D) AB and BA, both are not defined.

15. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(A) 3 (B) 7
(C) ± 7 (D) ± 3

16. If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1$, $a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(A) 0 (B) -10
(C) 10 (D) 1

17. If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is :

(A) 0 (B) 1
(C) -2 (D) -4

18. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(A) 9 (B) 512
(C) 615 (D) 64

19. The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an :

(A) scalar matrix
(B) identity matrix
(C) null matrix
(D) symmetric matrix

20. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30 (B) 120
(C) 15 (D) 225

21. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is :

(A) 5 (B) 125
(C) 25 (D) -5

22. If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x-y)$ is :

(A) 2 or 10
(B) -2 or 10
(C) 2 or -10
(D) -2 or -10

23. If the matrix $A = [a_{ij}]_{2\times 2}$ is such that $a_{ij} = \begin{cases} 1, \text{ if } i \neq j \\ 0, \text{ if } i = j \end{cases}$, then $A + A^2$ is equal to :

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

24. The value of the determinant $\begin{vmatrix} \cos 75^\circ & \sin 75^\circ \\ \sin 15^\circ & \cos 15^\circ \end{vmatrix}$ is :

(A) 1 (B) zero
(C) $\frac{1}{2}$ (D) $\frac{\sqrt{3}}{2}$

25. For a non-singular matrix X, if $X^2 = I$, then X^{-1} is equal to :

(A) X (B) X^2
(C) I (D) O

26. The cofactor of the element a_{32} in the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$ is :

(A) ± 5 (B) -5
(C) 5 (D) 0

27. If A is an identity matrix of order n, then $A(\text{Adj } A)$ is a/an :

(A) identity matrix
(B) row matrix
(C) zero matrix
(D) skew symmetric matrix

28. If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to

(A) 18 (B) 12
(C) 6 (D) 2

29. If $B = [23\ \ 41\ \ 57]\begin{bmatrix} 31 & 42 \\ 53 & 64 \\ 75 & 86 \end{bmatrix}$, then the order of B is :

(A) 3×2 (B) 2×2
(C) 1×3 (D) 1×2

30. If A and B are square matrices of the same order, then $(A - B)^2 =$?

(A) $A^2 - 2AB + B^2$
(B) $A^2 - AB - BA + B^2$
(C) $A^2 - 2BA + B^2$
(D) $A^2 - AB + BA + B^2$

31. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is

(A) 0 (B) 9
(C) -6 (D) 6

32. If $A^{-1} = \begin{bmatrix} 7 & 2 \\ 8 & 2 \end{bmatrix}$, then matrix A is

(A) $\begin{bmatrix} 2 & -2 \\ -8 & 7 \end{bmatrix}$
(B) $\begin{bmatrix} -7 & 8 \\ 2 & -2 \end{bmatrix}$
(C) $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$
(D) $\begin{bmatrix} 1 & -1 \\ -4 & \frac{7}{2} \end{bmatrix}$

33. If $A = \begin{bmatrix} x & 3 \\ 3 & x \end{bmatrix}$ and $|A^3| = 343$, then x is :

(A) ± 7 (B) ± 4
(C) ± 3 (D) ± 5

34. If A and B are square matrices both of order 3, such that $|A| = -3$ and $|B| = 2$, then $|2AB|$ is equal to :

(A) 48 (B) -48
(C) -24 (D) -12

35. For a skew-symmetric matrix $A = \begin{bmatrix} 0 & 7 & -4 \\ r & p & 5 \\ q & -5 & 0 \end{bmatrix}$, the value of $p + q - r$ is :

(A) 11 (B) -3
(C) -11 (D) 3

36. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 5 & 5 \\ 3 & 7 \end{vmatrix}$, then the value of x is :

(A) 4 (B) 3
(C) 6 (D) 2

37. If A is a square matrix of order 3 such that $A(\text{adj}\ \ A) = 7I$, then $|\text{adj}\ \ A|$ is equal to :

(A) 1 (B) 343
(C) 7 (D) 49

Assertion - Reason Questions (1 Mark)

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false but Reason (R) is true.

38. **Assertion (A):** $A = \text{diag}\ \ [3\ 5\ 2]$ is a scalar matrix of order 3×3.
Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

39. **Assertion (A):** If A is a skew-symmetric matrix of order 3, then $|A| = 0$.
Reason (R): If A is a square matrix of order 3, then $|A| = |A'|$.

Very Short Answer Questions (2 Marks)

40. Let A and B be two square matrices of order 3 such that $\det(A) = 3$ and $\det(B) = -4$. Find the value of $\det(-6AB)$.

41. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.

Short Answer & Long Answer Questions (3 - 5 Marks)

42. A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions :

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

43. (a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations: $x - y + z = 4$ $x - 2y - 2z = 9$ $2x + y + 3z = 1$

OR

(b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}. Hence, solve the system of linear equations: $x - 2y = 10$ $2x - y - z = 8$ $-2y + z = 7$

44. If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

45. A furniture workshop produces three types of furniture – chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45. It was also found that production of beds exceeds that of chairs by 8, while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.

46. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?

47. If $A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the given system of equations $3x + 4y + 7z = 14; 2x - y + 3z = 4; x + 2y - 3z = 0.$

48. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations : $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$

Case Study Based Questions (4 Marks)

49. To promote the making of toilets in villages, an NGO hired an agency for generating awareness for the cause through house calls, letters and announcements through speakers. The cost per mode of communication is given below :

Cost per visit / communication	House calls	Letters	Announcements
₹	15	10	25

The number of contacts made in two villages X and Y were as follows :

Village	Houses visited	Letters sent	Number of announcements
X	100	150	110
Y	150	200	150

Using the above information, answer the following questions : (i) Write A, the matrix for cost per visit/communication. **1** (ii) Write B, the matrix representing number of contacts in two villages, X and Y. **1** (iii) (a) Find the cost (in ₹) incurred by the NGO for village X. **2**

OR

(iii) (b) Find the cost (in ₹) incurred by the NGO for village Y. **2**

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Section A: Multiple Choice Questions (1 Mark)

1. If a matrix has 36 elements, the number of possible orders it can have, is :

(A) 13 (B) 3

(C) 5 (D) 9

2. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

(A) 7 (B) 6

(C) 8 (D) 18

3. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :

(A) $2x^3$ (B) 2

(C) 0 (D) $2x^3 - 2$

4. If A and B are two non-zero square matrices of same order such that $(A+B)^2 = A^2 + B^2$, then :

(A) $AB = O$

(B) $AB = -BA$

(C) $BA = O$

(D) $AB = BA$

5. If the sum of all the elements of a 3×3 scalar matrix is 9, then the product of all its elements is :

(A) 0 (B) 9

(C) 27 (D) 729

6. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

7. If $A = [a_{ij}]$ be a 3×3 matrix, where $a_{ij} = i - 3j$, then which of the following is **false** ?

(A) $a_{11} < 0$

(B) $a_{12} + a_{21} = -6$

(C) $a_{13} > a_{31}$

(D) $a_{31} = 0$

8. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :

(A) 1 (B) 2

(C) 0 (D) -2

9. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, \text{ if } i=j \\ 1, \text{ if } i \neq j \end{cases}$

(B) $a_{ij} = 1, \forall i, j$

(C) $a_{ij} = 0, \forall i, j$

(D) $a_{ij} = \begin{cases} 0, \text{ if } i \neq j \\ 1, \text{ if } i=j \end{cases}$

10. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj $A = A$. Then, $(a+b+c+d)$ is equal to :

(A) $2a$ (B) $2b$

(C) $2c$ (D) 0

11. If A and B are two skew symmetric matrices, then $(AB + BA)$ is :

(A) a skew symmetric matrix

(B) a symmetric matrix

(C) a null matrix

(D) an identity matrix

12. If $\begin{vmatrix} 1 & 3 & 1 \\ k & 0 & 1 \\ 0 & 0 & 1 \end{vmatrix} = \pm 6$, then the value of k is :

(A) 2 (B) -2

(C) ± 2 (D) ∓ 2

13. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(B) $30 \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(D) $\frac{1}{30} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

14. If $\begin{bmatrix} a & c & 0 \\ b & d & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is a scalar matrix, then the value of $a + 2b + 3c + 4d$ is :

(A) 0 (B) 5

(C) 10 (D) 25

15. Given that $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$, matrix A is :

(A) $7\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(C) $\frac{1}{7}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

(D) $\frac{1}{49}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$

16. If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + ...$ is :

(A) $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$
(B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$
(C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

17. If $A = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 2 & 3 \\ 5 & 1 & -1 \end{bmatrix}$, then the value of $|A(\text{adj } A)|$ is :
(A) $100I$ (B) $10I$
(C) 10 (D) 1000

18. Given that $\begin{bmatrix} 1 & x \end{bmatrix}\begin{bmatrix} 4 & 0 \\ -2 & 0 \end{bmatrix} = 0$, the value of x is :
(A) -4 (B) -2
(C) 2 (D) 4

19. If $A = \begin{bmatrix} a & c & -1 \\ b & 0 & 5 \\ 1 & -5 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of $2a - (b + c)$ is :
(A) 0 (B) 1
(C) -10 (D) 10

20. If A is a square matrix of order 3 such that the value of $|\text{adj } \cdot A| = 8$, then the value of $|A^T|$ is :
(A) $\sqrt{2}$ (B) $-\sqrt{2}$
(C) 8 (D) $2\sqrt{2}$

21. If inverse of matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 3 & 3 \\ 1 & \lambda & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then value of λ is :
(A) -4 (B) 1
(C) 3 (D) 4

22. If $\begin{bmatrix} x & 2 & 0 \end{bmatrix}\begin{bmatrix} 5 \\ -1 \\ x \end{bmatrix} = \begin{bmatrix} 3 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ x \end{bmatrix}$, then value of x is :
(A) -1 (B) 0
(C) 1 (D) 2

23. Find the matrix A^2, where $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \text{maximum } (i, j) - \text{minimum } (i, j)$:
(A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(D) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

24. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, then the value of x, for which A is an identity matrix, is
(A) $\frac{\pi}{2}$ (B) π
(C) 0 (D) $\frac{3\pi}{2}$

25. If the matrix $A = \begin{bmatrix} 0 & 5 & -7 \\ a & 0 & 3 \\ b & -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the values of 'a' and 'b' are :
(A) $a = 5, b = 3$
(B) $a = 5, b = -7$
(C) $a = -5, b = -7$
(D) $a = -5, b = 7$

26. If $\begin{vmatrix} x+2 & x-4 \\ x-2 & x+3 \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 1 & 3 \end{vmatrix}$, then the value of x is :
(A) 1 (B) 2
(C) -2 (D) -1

27. If $\begin{bmatrix} 8 & 14 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} X$, then matrix X is :
(A) $\begin{bmatrix} 3 & 7 \\ 2 & 0 \end{bmatrix}$
(B) $\begin{bmatrix} 2 & 0 \\ 7 & 3 \end{bmatrix}$
(C) $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$
(D) $\begin{bmatrix} 2 & 0 \\ -3 & 7 \end{bmatrix}$

28. If $A = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 2 & 1 \\ 0 & 3 & -2 \end{bmatrix}$, then the value of $|A \text{ adj } (A)|$ is :
(A) -1 (B) 1
(C) 2 (D) 3

29. For two matrices A and B, given that $A^{-1} = \frac{1}{4}B$, then inverse of $(4A)$ is :
(A) $4B$ (B) B
(C) $\frac{1}{4}B$ (D) $\frac{1}{16}B$

30. If X, Y and XY are matrices of order 2×3, $m \times n$ and 2×5 respectively, then number of elements in matrix Y is :
(A) 6 (B) 10
(C) 15 (D) 35

Section A: Assertion-Reason (1 Mark)

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct expla-

nation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

31. Assertion (A) : For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $|A| \in [2, 4]$.
Reason (R) : $\cos\theta \in [-1, 1], \forall\theta \in [0, 2\pi]$.

32. Assertion (A) : For any symmetric matrix A, $B'AB$ is a skew-symmetric matrix.
Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

33. Assertion (A) : Every scalar matrix is a diagonal matrix.
Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

Section C: Short Answer Type Questions (3 Marks)

34. (a) Find a matrix A such that $A\begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix}$. Also, find A^{-1}.

OR

(b) Given a square matrix A of order 3 such that $A^2 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$, show that $A^3 = A^{-1}$.

Section D: Long Answer Type Questions (5 Marks)

35. (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations : $x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

OR

(b) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$, find the value of $(a + x) - (b + y)$.

36. (a) Solve the following system of equations, using matrices : $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4, \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1, \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$ where $x, y, z \neq 0$

OR

(b) If $A = \begin{bmatrix} 1 & \cot x \\ -\cot x & 1 \end{bmatrix}$, show that $A'A^{-1} = \begin{bmatrix} -\cos 2x & -\sin 2x \\ \sin 2x & -\cos 2x \end{bmatrix}$.

37. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 2 & 1 \\ 2 & 1 & -3 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations : $2x + y - 3z = 13$ $3x + 2y + z = 4$ $x + 2y - z = 8$

38. (a) If $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & -3 \\ 1 & 2 & 0 \end{bmatrix}$, then find A^{-1} and hence solve the following system of equations : $x + 2y - 3z = 1$ $2x - 3z = 2$ $x + 2y = 3$

OR

(b) Find the product of the matrices $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$ and hence solve the system of linear equations : $x + 2y - 3z = -4$ $2x + 3y + 2z = 2$ $3x - 3y - 4z = 11$

39. Using matrices, solve the following system of linear equations : $3x + 4y + 2z = 8$; $2y - 3z = 3$; $x - 2y + 6z = -2$

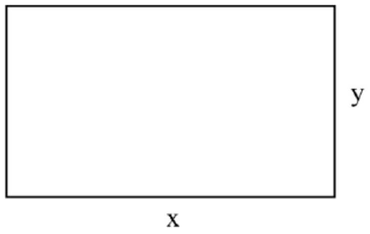
Section E: Case Study Based Questions (4 Marks)

40. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs. Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports. In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000. Based on the above information, answer the following questions : (i) Express the given information algebraically using matrices. (1) (ii) Check whether the system of matrix equations so obtained is consistent or not. (1) (iii) (a) Find the number of scholarships of each kind given by the school, using matrices. (2)

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? (2)

41. An architect is developing a plot of land for a commercial complex. When asked about the dimensions of the plot, he said that if the length is decreased by 25 m and the breadth is increased by 25 m, then its area increases by 625 m^2. If the length is decreased by 20 m and the breadth is increased by 10 m, then its area decreases by 200 m^2.



On the basis of the above information, answer the following questions : (i) Formulate the linear equations in x and y to represent the given information. (ii) Find the dimensions of the plot of land by matrix method.

5.CONTINUITY & DIFFERENTIABILITY

March, 2025

Multiple Choice Questions (1 Mark)

1. If $f(x) = |x| + |x-1|$, then which of the following is correct ?
(A) $f(x)$ is both continuous and differentiable, at $x=0$ and $x=1$.
(B) $f(x)$ is differentiable but not continuous, at $x=0$ and $x=1$.
(C) $f(x)$ is continuous but not differentiable, at $x=0$ and $x=1$.
(D) $f(x)$ is neither continuous nor differentiable, at $x=0$ and $x=1$.

2. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B?

3. If $y = \sin^{-1} x$, then $(1-x^2)\frac{d^2y}{dx^2}$ is equal to :
(A) $x\frac{dy}{dx}$
(B) $-x\frac{dy}{dx}$
(C) $x^2\frac{dy}{dx}$
(D) $-x^2\frac{dy}{dx}$

4. If $f(x) = \begin{cases} 3x-2, & 0<x\leq 1 \\ 2x^2+ax, & 1<x<2 \end{cases}$ is continuous for $x \in (0,2)$, then a is equal to :
(A) – 4
(B) $-\frac{7}{2}$
(C) – 2
(D) – 1

5. The function f defined by $f(x) = \begin{cases} x, & \text{if } x\leq 1 \\ 5, & \text{if } x>1 \end{cases}$ is **not** continuous at :
(A) x = 0
(B) x = 1
(C) x = 2
(D) x = 5

6. If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x\neq 0 \\ 1, & x=0 \end{cases}$ is continuous at x = 0, then the value of a is :
(A) 1
(B) – 1
(C) ±1
(D) 0

7. If $f(x) = \{[x], x \in \mathbb{R}\}$ is the greatest integer function, then the correct statement is :
(A) f is continuous but not differentiable at x = 2.
(B) f is neither continuous nor differentiable at x = 2.
(C) f is continuous as well as differentiable at x = 2.
(D) f is not continuous but differentiable at x = 2.

8. If $f(x) = \begin{cases} 1, & \text{if } x\leq 3 \\ ax+b, & \text{if } 3<x<5 \\ 7, & \text{if } 5\leq x \end{cases}$ is continuous in $\mathbb{R}$, then the values of a and b are :
(A) $a=3, b=-8$
(B) $a=3, b=8$
(C) $a=-3, b=-8$
(D) $a=-3, b=8$

9. If $f(x) = -2x^8$, then the correct statement is :
(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$
(B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$
(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$
(D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

10. If $x = t^3$ and $y = t^2$, then $\frac{d^2y}{dx^2}$ at $t=1$ is :
(A) $\frac{3}{2}$
(B) $-\frac{2}{9}$
(C) $-\frac{3}{2}$
(D) $-\frac{2}{3}$

11. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is
(A) $\frac{-\sqrt{x}}{\sqrt{y}}$
(B) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$
(C) $\frac{-\sqrt{y}}{\sqrt{x}}$
(D) $\frac{-2\sqrt{y}}{\sqrt{x}}$

12. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin}x\right)$, then $\frac{dy}{dx}$ is
(A) 1
(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$
(D) – 1

13. If $y = \log \cos^2 \sqrt{x}$, then $\frac{dy}{dx}$ is :
(A) $\frac{\tan \sqrt{x}}{\sqrt{x}}$
(B) $2 \tan \sqrt{x}$
(C) $\frac{-\tan \sqrt{x}}{\sqrt{x}}$
(D) $\frac{-\sqrt{x}}{\tan} \sqrt{x}$

14. If $f(x) = |x - 1|$, then f'(1) :
(A) is – 1
(B) is + 1
(C) is 0
(D) does not exist

Assertion - Reason Questions (1 Mark)

15. Assertion (A) : $f(x) = \begin{cases} 3x-8 \ , & x \leq 5 \\ 2k \ , & x>5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.
Reason (R) : For a function f to be continuous at $x = a$, $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

16. Assertion (A) : $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right) \ , & x \neq 0 \\ 0 \ , & x=0 \end{cases}$ is continuous at $x = 0$.
Reason (R) : When $x \to 0, \sin\left(\frac{1}{x}\right)$ is a finite value between -1 and 1.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

17. Assertion (A) : $f(x) = [x], x \in \mathbb{R}$, the greatest integer function is not differentiable at $x = 2$.
Reason (R) : The greatest integer function is not continuous at any integral value.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Very Short Answer Questions (2 Marks)

18. (a) Differentiate $2\cos^2 x$ w.r.t $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

19. (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x \log x}$.

OR

(b) If $f(x) = \begin{cases} 2x-3 \ , & -3 \leq x \leq -2 \\ x+1 \ , & -2 < x \leq 0 \end{cases}$ Check the differentiability of $f(x)$ at $x = -2$.

20. (a) Find k so that $f(x) = \begin{cases} \frac{x^2-2x-3}{x+1} \ , & x \neq -1 \\ k \ , & x=-1 \end{cases}$ is continuous at x = – 1.

OR

(b) Check the differentiability of function $f(x) = x|x|$ at x = 0.

21. (a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x.

OR

(b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

22. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

23. If $e^y(x + 1) = 1$, prove that $\frac{dy}{dx} = -e^y$.

24. (a) Show that the function $f(x) = (x - 1)^{\frac{1}{3}}$ is not differentiable at $x = 1$.

OR

(b) Differentiate $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x.

25. (a) Find the value of k, so that $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

OR

(b) Find $\frac{dy}{dx}$, if $y = \tan^{-1}\left(\frac{1+\sin x}{\cos}x\right)$.

Short Answer & Long Answer Questions (3 - 5 Marks)

26. (a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

27. (a) For a positive constant 'a', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t + \frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

28. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.

29. (a) If $y = \cos x^2 + \cos^2 x + \cos^2(x^2) + \cos(x^x)$, find $\frac{dy}{dx}$.

30. (a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x.

OR

(b) If $y = x + \tan x$, then prove that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$

31. (a) If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right], x^2 \leq 1$, then find $\frac{dy}{dx}$.

OR

(b) If $y = \left(x + \sqrt{1+x^2}\right)^n$, then show that $(1+x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2 y$.

March, 2024

Section A: Multiple Choice Questions (1 Mark)

1. Which of the following statements is true for the function $f(x) = \begin{cases} x^2+3, & x \neq 0 \\ 1, & x = 0 \end{cases}$?

(A) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$

(B) f(x) is continuous $\forall x \in \mathbb{R}$

(C) f(x) is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$

(D) f(x) is discontinuous at infinitely many points

2. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is :

(A) 1

(B) – 1

(C) $-2\sqrt{\pi}$

(D) $2\sqrt{\pi}$

3. The number of points of discontinuity of $f(x) = \begin{cases} |x| + 3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3 < x < 3 \\ 6x + 2, & \text{if } x \geq 3 \end{cases}$ is :

(A) 0

(B) 1

(C) 2

(D) infinite

4. The derivative of $\tan^{-1}(x^2)$ w.r.t. x is :

(A) $\frac{x}{1+x^4}$

(B) $\frac{2x}{1+x^4}$

(C) $\frac{-2x}{1+x^4}$

(D) $\frac{1}{1+x^4}$

5. A function $f(x) = |1 - x + |x\|$ is :

(A) discontinuous at x = 1 only

(B) discontinuous at x = 0 only

(C) discontinuous at x = 0, 1

(D) continuous everywhere

6. The derivative of 2^x w.r.t. 3^x is :

(A) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$

(B) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$

(C) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

(D) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$

7. Derivative of e^{2x} with respect to e^x, is :

(A) e^x

(B) $2e^x$

(C) $2e^{2x}$

(D) $2e^{3x}$

8. For what value of k, the function given below is continuous at $x = 0$?

$f(x) = \begin{cases} \frac{\sqrt{4+x}-2}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$

(A) 0

(B) $\frac{1}{4}$
(C) 1
(D) 4

9. If $xe^y = 1$, then the value of $\frac{dy}{dx}$ at $x = 1$ is :
(A) -1
(B) 1
(C) $-e$
(D) $-\frac{1}{e}$

10. Derivative of $e^{\sin^2 x}$ with respect to $\cos x$ is :
(A) $\sin x \cdot e^{\sin^2 x}$
(B) $\cos x \cdot e^{\sin^2 x}$
(C) $-2\cos x \cdot e^{\sin^2 x}$
(D) $-2\sin^2 x \cos x \cdot e^{\sin^2 x}$

11. The value of k, for which $f(x) = \begin{cases} \frac{\sqrt{3}\cos x + \sin x}{3x+\pi}, & x \neq -\frac{\pi}{3} \\ k, & x = -\frac{\pi}{3} \end{cases}$ is continuous at $x = -\frac{\pi}{3}$, is :
(A) $\frac{2}{3}$
(B) $-\frac{2}{3}$
(C) $\frac{3}{2}$
(D) 6

12. The number of discontinuities of the function f given by $f(x) = \begin{cases} x+2, & \text{if } x<0 \\ e^x, & \text{if } 0 \leq x \leq 1 \\ 2-x, & \text{if } x>1 \end{cases}$ is :
(A) 0
(B) 1
(C) 2
(D) 3

13. Let $y = f\left(\frac{1}{x}\right)$ and $f'(x) = x^3$. What is the value of $\frac{dy}{dx}$ at $x = \frac{1}{2}$?
(A) $-\frac{1}{64}$
(B) $-\frac{1}{32}$
(C) – 32
(D) – 64

14. If $y = \log\sqrt{\sec\sqrt{x}}$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi^2}{16}$ is :
(A) $\frac{1}{\pi}$
(B) π
(C) $\frac{1}{2}$
(D) $\frac{1}{4}$

15. If $x = 3\cos\theta$ and $y = 5\sin\theta$, then $\frac{dy}{dx}$ is equal to :
(A) $-\frac{3}{5}\tan\theta$
(B) $-\frac{5}{3}\cot\theta$
(C) $-\frac{5}{3}\tan\theta$
(D) $-\frac{3}{5}\cot\theta$

16. The greatest integer function defined by f(x) = [x], 1 < x < 3 is not differentiable at x =
(A) 0
(B) 1
(C) 2
(D) $\frac{3}{2}$

Section B: Very Short Answer Type Questions (2 Marks)

17. (a) Check whether the function $f(x) = x^2\ |x|$ is differentiable at x = 0 or not.

OR

(b) If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$.

18. (a) If $f(x) = |\tan 2x|$, then find the value of f'(x) at $x = \frac{\pi}{3}$.

OR

(b) If $y = \text{cosec}\left(\cot^{-1} x\right)$, then prove that $(1+x^2)\frac{dy}{dx} - x = 0$.

19. (a) If $x = e^{\frac{x}{y}}$, prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$?

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2+1, & 0 \leq x < 1 \\ 3-x, & 1 \leq x \leq 2 \end{cases}$ at x = 1.

20. (a) If $y = \cos^3(\sec^2 2t)$, find $\frac{dy}{dt}$.

OR

(b) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

21. (a) Verify whether the function f defined by $f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous at $x = 0$ or not.

OR

(b) Check for differentiability of the function f defined by $f(x) = |x-5|$, at the point $x = 5$.

22. (a) Differentiate $\cot^{-1}\left(\sqrt{1+x^2} + x\right)$ w.r.t. x.

OR

(b) If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$.

23. (a) If $y = \left(\sin^{-1} x\right)^2$, then find $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$.

OR

(b) If $y^x = x^y$, then find $\frac{dy}{dx}$.

Section C: Short Answer Type Questions (3 Marks)

24. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

25. (a) If $x = e^{\cos 3t}$ and $y = e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{y \log x}{x \log y}$.

OR

(b) Show that : $\frac{d}{dx}(|x|) = \frac{x}{|}x|, x \neq 0$

26. (a) If $x \cos(p+y) + \cos p \sin(p+y) = 0$, prove that $\cos p \frac{dy}{dx} = -\cos^2(p+y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as : $f(x) = \begin{cases} \frac{|x-2|}{x-2}+a \ , & \text{if } \ x<2 \\ a+b \ , & \text{if } \ x=2 \\ \frac{x-2}{|}x-2| +b \ , & \text{if } \ x>2 \end{cases}$ is a continuous function.

27. Given that $y = (\sin x)^x + x^{\sin x} + a^x$, find $\frac{dy}{dx}$.

28. (a) Find $\frac{dy}{dx}$, if $(\cos x)^y = (\cos y)^x$.

OR

(b) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

29. If $x = a \sin^3 \theta, y = b \cos^3 \theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

30. If $x = a \cos \theta$ and $y = b \sin \theta$, then prove that $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2 y^3}$.

31. (a) If $x \sin(a+y) - \sin y = 0$, prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

OR

(b) Find $\frac{dy}{dx}$, if $y = (\cos x)^x + \cos^{-1} \sqrt{x}$.

6. APPLICATIONS OF DERIVATIVES

March, 2025

Multiple Choice Questions (1 Mark)

1. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is :
(A) 0
(B) 2
(C) 4
(D) 5

2. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval
(A) $(0, 2)$
(B) $(-\infty, 2]$
(C) $[1, 2]$
(D) $[2, \infty)$

3. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of $100\pi \frac{\text{cm}^3}{\text{s}}$. The rate, at which the height of the sugar inside the tank is increasing, is:
(A) 0.1 cm/s
(B) 0.5 cm/s
(C) 1 cm/s
(D) 1.1 cm/s

4. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :
(A) $1 < \lambda < \sqrt{2}$
(B) $\lambda \geq 1$
(C) $\lambda \geq \sqrt{2}$
(D) $\lambda < 1$

5. If $f(x) = 2x + \cos x$, then $f(x)$:
(A) has a maxima at x = π
(B) has a minima at x = π
(C) is an increasing function
(D) is a decreasing function

6. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :
(A) (1, –10)
(B) (1, 10)
(C) (10, 1)
(D) (–10, 1)

7. A spherical ball has a variable diameter $\frac{5}{2}(3x + 1)$. The rate of change of its volume w.r.t. x, when $x = 1$, is :
(A) 225π
(B) 300π
(C) 375π
(D) 125π

8. If $f : \mathbb{R} \to \mathbb{R}$ is defined as $f(x) = 2x - \sin x$, then f is :
(A) a decreasing function
(B) an increasing function
(C) maximum at $x = \frac{\pi}{2}$
(D) maximum at $x = 0$

9. If the rate of change of volume of a sphere is twice the rate of change of its radius, then the surface area of the sphere is :
(A) 1 sq unit
(B) 2 sq units
(C) 3 sq units
(D) 4 sq units

10. When x is positive, the minimum value of x^x is
(A) e^e
(B) $\frac{1}{e}$
(C) e^e
(D) $e^{-\frac{1}{e}}$

11. If $f(x) = x^2 + ax + 3$ is strictly increasing in the interval (3, 4), then the minimum value of a is :
(A) – 6
(B) – 8
(C) 6
(D) 8

Very Short Answer Questions (2 Marks)

12. Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.

13. Find the values of ‘a’ for which $f(x) = \sin x - ax + b$ is increasing on bb(R).

14. (a) Find the least value of ‘a’ so that $f(x) = 2x^2 - ax + 3$ is an increasing function on [2, 4].

OR

(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

15. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when x = 2 ?

16. Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm, find the rate at which the volume of the balloon is increasing.

17. A ladder 13 m long is leaning against the wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 2 m/s. How fast is the height on the wall decreasing when the foot of the ladder is 12 m away from the wall ?

18. If $y = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?

19. The radius of a cylinder is increasing at the rate of 3 cm/s, and its height is decreasing at the rate of 5 cm/s. Find the rate of change of its volume, when radius is 4 cm and height is 7 cm.

Short Answer & Long Answer Questions (3 - 5 Marks)

20. The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?

21. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on [1, 5].

22. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.
(i) Find the rate of growth of the plant with respect to sunlight.
(ii) In how many days will the plant attain its maximum height ? What is the maximum height ?

23. Find the value of 'a' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in bb(R).

24. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

25. Find the maximum slope of the curve $y = -x^3 + 3x^2 + 9x - 30$.

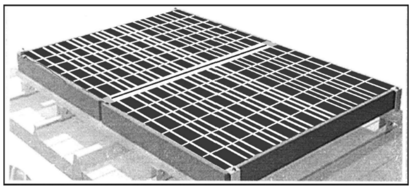
26. (b) Find the intervals in which the function given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is :
(i) strictly increasing.
(ii) strictly decreasing.

27. Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is
(a) strictly increasing
(b) strictly decreasing

28. Find the intervals in which the function $f(x) = \frac{3}{2}x^4 - 4x^3 - 45x^2 + 51$ is :
(a) strictly increasing.
(b) strictly decreasing.

Case Study Based Questions (4 Marks)

29.



A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections. Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.

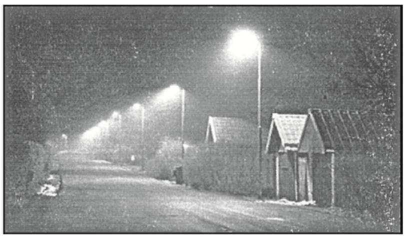
Based on this information, answer the following questions :
(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.
(ii) Write the area of the solar panel as a function of x.
(iii) (a) Find the critical points of the area function. Use second derivative test to deter-

mine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

30.



A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres. Based on the above, answer the following:

(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$.

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion.

31. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum. On the basis of the above information, answer the following questions :

(i) Taking length = breadth = x m and height = y m, express the surface area (S) of the box in terms of x and its volume (V), which is constant.

(ii) Find $\frac{dS}{dx}$.

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum.

OR

(iii) (b) If surface area (S) is constant, the volume (V) $= \frac{1}{4}(Sx - 2x^3)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$.

32. A window is in the form of a rectangle surmounted by an equilateral triangle on its length. Let the rectangular part have length and breadth x and y metres respectively. Based on the given information, answer the following questions :

(i) If the perimeter of the window is 12 m, find the relation between x and y.

(ii) Using the expression obtained in (i), write an expression for the area of the window as a function of x only.

(iii) (a) Find the dimensions of the rectangle that will allow maximum light through the window. (use expression obtained in (ii))

OR

(iii) (b) If it is given that the area of the window is $50m^2$, find an expression for its perimeter in terms of x.

33. An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m. Based on the above information, answer the following :

(i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y.

(ii) Find the total area of the window in terms of x.

(iii) (a) Find the values of x and y for the maximum area of the window.

OR

(iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.

34. A magazine company circulates its magazine on a monthly basis in a city. It has 10,000 readers on its list and collects fixed charges of ₹ 4,000 per reader annually. The company proposes to increase the annual subscription, but on the basis of a survey result, it predicted that for every increase of ₹ 5, ten readers will discontinue the service of this magazine company. Based on the above information, answer the following questions :

(i) Let the company increase ₹ x, then find the function R(x) representing the earnings of the company.

(ii) Find $\frac{d}{dx}(R(x))$.

(iii) (a) What subscription increase will bring maximum earnings for the company ?

OR

(iii) (b) What will be the maximum value of R(x) ?

March, 2024

Section A: Multiple Choice Questions (1 Mark)

1. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if
(A) $f'(x) < 0, \forall x \in (a, b)$
(B) $f'(x) > 0, \forall x \in (a, b)$
(C) $f'(x) = 0, \forall x \in (a, b)$
(D) $f(x) > 0, \forall x \in (a, b)$

2. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :
(A) strictly decreasing on bb(R)
(B) strictly increasing on bb(R)
(C) neither strictly increasing nor strictly decreasing on bb(R)
(D) strictly decreasing on $(-\infty, 0)$

3. If the sides of a square are decreasing at the rate of 1.5 cm/s, the rate of decrease of its perimeter is :
(A) 1.5 cm/s
(B) 6 cm/s
(C) 3 cm/s
(D) 2.25 cm/s

4. The function $f(x) = kx - \sin x$ is strictly increasing for
(A) $k > 1$
(B) $k < 1$
(C) $k > -1$
(D) $k < -1$

5. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minima at x equal to :
(A) 2
(B) 1
(C) 0
(D) -2

6. Given a curve $y = 7x - x^3$ and x increases at the rate of 2 units per second. The rate at which the slope of the curve is changing, when $x = 5$ is :
(A) -60 units/sec
(B) 60 units/sec
(C) -70 units/sec
(D) -140 units/sec

7. For the function $f(x) = x^3$, x = 0 is a point of :
(A) local maxima
(B) local minima
(C) non-differentiability
(D) inflexion

8. If the radius of a circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is :
(A) $\frac{2\pi}{3}$ cm/s
(B) π cm/s
(C) $\frac{4\pi}{3}$ cm/s
(D) 2π cm/s

Section A: Assertion-Reason (1 Mark)

9. **Assertion (A) :** If the side of a square is increasing at the rate of 0.2 cm/s, then the rate of increase of its perimeter is 0.8 cm/s.
Reason (R) : Perimeter of a square = 4 (side).
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B: Very Short Answer Type Questions (2 Marks)

10. Show that the function $f(x) = 4x^3 - 18x^2 + 27x - 7$ has neither maxima nor minima.

11. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x} (x \neq 0)$ respectively, find the value of (M – m).

12. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

13. Find the interval in which the function $f(x) = x^4 - 4x^3 + 10$ is strictly decreasing.

14. The volume of a cube is increasing at the rate of 6 cm^3/s. How fast is the surface area of cube increasing, when the length of an edge is 8 cm ?

15. The area of the circle is increasing at a uniform rate of 2 cm^2/sec. How fast is the circumference of the circle increasing when the radius $r = 5$ cm ?

16. Find the intervals on which the function $f(x) = 10 - 6x - 2x^2$ is
(a) strictly increasing (b) strictly decreasing.

17. Show that of all rectangles inscribed in a given circle, the square has the maximum area.

18. Given that $f(x) = \frac{\log x}{x}$, find the point of local maximum of f(x).

Section C: Short Answer Type Questions (3 Marks)

19. (a) Find the intervals in which the function $f(x) = \frac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \frac{x}{2} + \frac{2}{x}$, on the interval [1, 2].

Section D: Long Answer Type Questions (5 Marks)

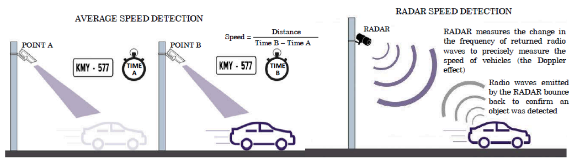
20. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

Section E: Case Study Based Questions (4 Marks)

21. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



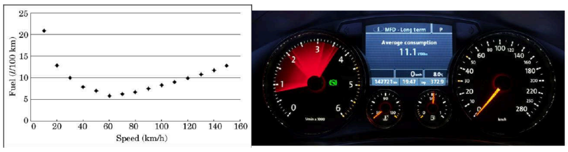
A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :
(i) Express θ in terms of height of the camera installed on the pole and x.
(ii) Find $\frac{d\theta}{dx}$.
(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car.

22. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



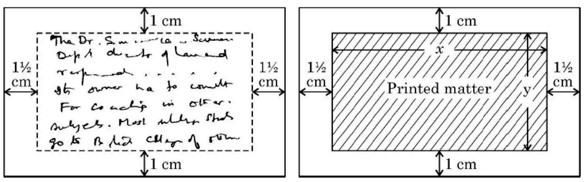
The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \frac{V^2}{500} - \frac{V}{4} + 14$.
On the basis of the above information, answer the following questions :
(i) Find F, when V = 40 km/h.
(ii) Find $\frac{dF}{dV}$.
(iii) (a) Find the speed V for which fuel consumption F is minimum.

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\frac{dF}{dV} = -0.01$.

23. A store has been selling calculators at ₹ 350 each. A market survey indicates that a reduction in price (p) of calculator increases the number of units (x) sold. The relation between the price and quantity sold is given by the demand function $p = 450 - \frac{1}{2}x$.
Based on the above information, answer the following questions :
(i) Determine the number of units (x) that should be sold to maximise the revenue $R(x) = xp(x)$. Also, verify the result.
(ii) What rebate in price of calculator should the store give to maximise the revenue ?

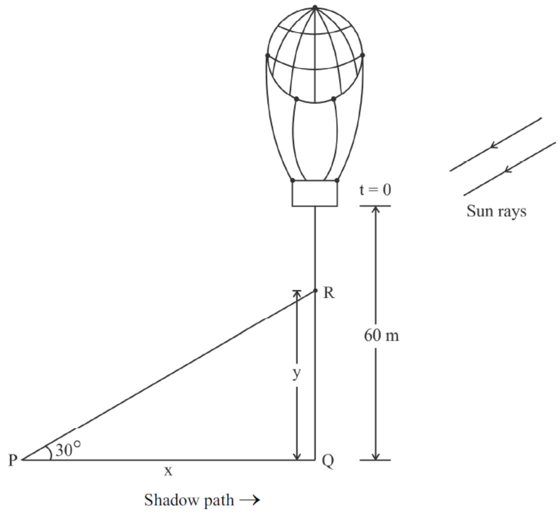
24. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1.5 cm as shown below :



On the basis of the above information, answer the following questions :
(i) Write the expression for the area of the visiting card in terms of x.
(ii) Obtain the dimensions of the card of minimum area.

25. The relation between the height of the plant (y in cm) with respect to exposure to sunlight is governed by the relation $y = 4x - \frac{1}{2}x^2$, where x is the number of days it is exposed to sunlight.
Based on the above, answer the following questions :
(i) Find the rate of growth of the plant with respect to sunlight.
(ii) What is the number of days it will take for the plant to grow to the maximum height ?
(iii) What is the maximum height of the plant ?

26. A sandbag is dropped from a balloon at a height of 60 metres.



When the angle of elevation of the sun is 30°, the position of the sandbag is given by the equation $y = 60 - 4.9t^2$, where y is the height of the sandbag above the ground and t is the time in seconds.
On the basis of the above information, answer the following questions :
(i) Find the relation between x and y, where x is the distance of the shadow at P from the point Q and y is the height of the sandbag above the ground.
(ii) After how much time will the sandbag be 35 metres above the ground ?
(iii) (a) Find the rate at which the shadow of the sandbag is travelling along the ground when the sandbag is at a height of 35 metres.

OR

(iii) (b) How fast is the height of the sandbag decreasing when 2 seconds have elapsed ?

7. INTEGRALS

March, 2025

Multiple Choice Questions (1 Mark)

1. $\int_{-1}^{1} \frac{|x|}{x} dx, x \neq 0$ is equal to:

(A) –1 (B) 0
(C) 1 (D) 2

2. If $\int \frac{2^{\frac{1}{x}}}{x^2} dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to:

(A) $-\frac{1}{\log 2}$ (B) $-\log 2$
(C) –1 (D) $\frac{1}{2}$

3. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)\, dx$ is:

(A) $\int_0^{2a} f\left(\frac{x}{2}\right) dx$
(B) $\int_0^{a} f(x)\, dx$
(C) $2\int_a^{0} f(x)\, dx$
(D) $2\int_0^{a} f(x)\, dx$

4. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to:

(A) $2(\sin x + x\cos\alpha) + C$
(B) $2(\sin x - x\cos\alpha) + C$
(C) $2(\sin x + 2x\cos\alpha) + C$
(D) $2(\sin x + \sin\alpha) + C$

5. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is:

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$
(C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$

6. $\int \sqrt{1 + \sin x}\, dx$ is equal to:

(A) $2\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$
(B) $2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$
(C) $-2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$
(D) $2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

7. $\int_0^{\frac{\pi}{2}} \cos x \cdot e^{\sin x}\, dx$ is equal to:

(A) 0 (B) 1 – e
(C) e – 1 (D) e

8. $\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}$ dx is equal to:

(A) $x + \text{C}$ (B) $\frac{x^2}{2} + \text{C}$
(C) $\frac{x^4}{4} + \text{C}$ (D) $\frac{x^3}{3} + \text{C}$

9. For a function f(x), which of the following holds true?

(A) $\int_a^b \text{f}(x)\ \text{dx} = \int_a^b \text{f}(a + b - x)\ \text{dx}$
(B) $\int_{-a}^{a} \text{f}(x)\ \text{dx} = 0$, if f is an even function
(C) $\int_{-a}^{a} \text{f}(x)\ \text{dx} = 2\int_0^a \text{f}(x)\ \text{dx}$, if f is odd
(D) $\int_0^{2a} \text{f}(x)\ \text{dx} = \int_0^a \text{f}(x)\ \text{dx} - \int_0^a \text{f}(2a + x)\ \text{dx}$

10. $\int \frac{e^x}{\sqrt{4 - e^{2x}}}$ dx is equal to:

(A) $\frac{1}{2}\cos^{-1}(e^x) + \text{C}$
(B) $\frac{1}{2}\sin^{-1}(e^x) + \text{C}$
(C) $\frac{e^x}{2} + \text{C}$
(D) $\sin^{-1}\left(\frac{e^x}{2}\right) + \text{C}$

11. $\int \frac{3\cos\sqrt{x}}{\sqrt{x}} dx$ is equal to:

(A) $-6\sin\sqrt{x} + C$
(B) $-6\cos\sqrt{x} + C$
(C) $6\cos\sqrt{x} + C$
(D) $6\sin\sqrt{x} + C$

12. $\int \frac{2x^3}{4 + x^8}$ dx is equal to:

(A) $\frac{1}{4}\tan^{-1}\frac{x^4}{2} + \text{C}$
(B) $\frac{1}{2}\tan^{-1}\frac{x^4}{2} + \text{C}$
(C) $\frac{1}{4}\tan^{-1}\frac{x^4}{4} + \text{C}$
(D) $\frac{1}{4}\tan^{-1} x^4 + \text{C}$

13. $\int \text{e}^x \cdot \frac{x}{(1+x)^2}$ dx is equal to:

(A) $\text{e}^x \cdot \frac{x}{1+x} + \text{C}$
(B) $\text{e}^x \cdot \frac{1}{1+x} + \text{C}$
(C) $\text{e}^x \cdot \frac{1}{x} + \text{C}$
(D) $\text{e}^x \cdot \frac{1}{(1+x)^2} + \text{C}$

14. $\int \frac{1}{9x^2+6x+10}dx$ is equal to:

(A) $\frac{1}{3}\tan^{-1}(3x+1)+C$
(B) $\frac{1}{9}\tan^{-1}(3x+1)+C$
(C) $\frac{\tan^{-1}}{3}\frac{3x+1}{3}+C$
(D) $\frac{1}{9}\tan^{-1}\frac{3x+1}{3}+C$

15. The value of $\int_{-2}^{2}\sin^5 x\cos x dx$ is:

(A) $\frac{64}{3}$ (B) 0
(C) $2\sin^6 2$ (D) $\sin^6(-2)-\sin^6 2$

Very Short Answer Questions (2 Marks)

16. Evaluate: $\int_0^{\frac{\pi}{4}}\sqrt{1+\sin 2x}\,dx$

Short Answer & Long Answer Questions (3 - 5 Marks)

17. (a) Find : $\int \frac{x+\sin x}{1+\cos x}dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

18. Find: $\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\,dx$.

19. Find: $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$.

20. Evaluate : $\int_{\frac{\pi}{2}}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

21. (a) Find : $\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}dx$

OR

(b) Evaluate : $\int_0^{\pi}\frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$

22. (a) Find : $\int \frac{2x}{(x^2+3)(x^2-5)}dx$

OR

(b) Evaluate : $\int_1^4(|x-2|+|x-4|)\,dx$

23. (a) Find : $\int \frac{x^2+1}{(x^2+2)(2x^2+1)}$ dx

OR

(b) Evaluate : $\int_0^{\pi}\frac{x\tan x}{\sec x+\tan x}$ dx

24. (a) Find : $\int \sqrt{4x^2-4x+10}dx$

OR

(b) Evaluate : $\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx$

25. (a) Find : $\int \frac{x^2-x+1}{(x-1)(x^2+1)}$ dx

OR

(b) Evaluate : $\int_1^4(|x|+|3-x|)$ dx

26. (a) Find : $\int \sin^3 x\cdot\cos^4 x dx$

OR

(b) Evaluate : $\int_{-2}^{1}|x^3-x|\;dx$

MARCH, 2024

Section A: Multiple Choice Questions (1 Mark)

1. $\int_a^b f(x)dx$ is equal to:

(A) $\int_a^b f(a-x)dx$
(B) $\int_a^b f(a+b-x)dx$
(C) $\int_a^b f(x-(a+b))dx$
(D) $\int_a^b f((a-x)+(b-x))dx$

2. $\int_0^{\frac{\pi}{2}}\frac{\sin x-\cos x}{1+\sin x\cos x}dx$ is equal to:

(A) π
(B) Zero (0)
(C) $\int_0^{\frac{\pi}{2}}\frac{2\sin x}{1+\sin x\cos x}dx$
(D) $\frac{\pi^2}{4}$

3. $\int_{-a}^{a} f(x)dx=0$, if:

(A) f(– x) = f(x) (B) f(– x) = – f(x)
(C) f(a – x) = f(x) (D) f(a – x) = – f(x)

4. The value of $\int_0^3\frac{dx}{\sqrt{9-x^2}}$ is:

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{18}$

5. $\int \frac{1}{x(\log x)^2}dx$ is equal to:

(A) $2\log(\log x)+c$
(B) $-\frac{1}{\log x}+c$
(C) (log x)^3/3 + c
(D) 3/(log x)^3 + c

6. The value of $\int_{-1}^{1} x\ |x|\ dx$ is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$
(C) $-\frac{1}{6}$ (D) 0

7. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to:

(A) $\cot x + \tan x + c$
(B) $-\cot x + \tan x + c$
(C) $\cot x - \tan x + c$
(D) $-\cot x - \tan x + c$

8. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} x^3 \cos^2 x dx$ is equal to:

(A) 0 (B) –1
(C) 1 (D) 2

9. $\int \frac{x-3}{(x-1)^3} e^x dx$ is equal to:

(A) $\frac{2e^x}{(x-1)^3} + C$
(B) $\frac{-2e^x}{(x-1)^2} + C$
(C) $\frac{e^x}{x-1} + C$
(D) $\frac{e^x}{(x-1)^2} + C$

Section B: Very Short Answer Type Questions (2 Marks)

10. (a) Find : $\int x\sqrt{1+2x}dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi^2}{4}} \frac{\sin\sqrt{x}}{\sqrt{x}} dx$

11. Find : $\int \frac{e^{4x}-1}{e^{4x}+1} dx$

12. (a) Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \cos 3x dx$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x-x^2}}$ and F(1) = 0, find F(x).

13. Find : $\int \frac{1}{x(x^2-1)} dx$.

14. (a) Find : $\int \cos^3 x\ e^{\log \sin x} dx$

OR

(b) Find : $\int \frac{1}{5+4x-x^2} dx$

15. Find : $\int \csc^3(3x+1)\cot(3x+1)dx$

16. (a) Find : $\int \frac{x^3-1}{x^3-x} dx$

OR

(b) Evaluate : $\int_{-4}^{0} |x+2|\ dx$

Section C: Short Answer Type Questions (3 Marks)

17. (a) Find : $\int \frac{x^2}{(x^2+4)(x^2+9)} dx$

OR

(b) Evaluate : $\int_1^3 (|x-1| + |x-2| + |x-3|) dx$

18. (a) Evaluate : $\int_{-2}^{2} \frac{x^2}{\sqrt{2+x}} dx$

OR

(b) Find : $\int \frac{1}{x[(\log x)^2 - 3\log x - 4]} dx$

19. Find : $\int x^2 \cdot \sin^{-1}\left(x^{\frac{3}{2}}\right) dx$

20. Find : $\int \frac{x^2+1}{(x^2+2)(x^2+4)} dx$

21. (a) Find : $\int e^x \frac{2+\sin 2x}{1+\cos 2x} dx$

OR

(b) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{1}{\sin x + \cos x} dx$

22. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{x\ dx}{1+\cos 2x + \sin 2x}$

OR

(b) Find : $\int e^x \left[\frac{1}{(1+x^2)^{\frac{3}{2}}} + \frac{x}{\sqrt{1+x^2}}\right] dx$

23. Find : $\int \frac{3x+5}{\sqrt{x^2+2x+4}} dx$

24. (a) Evaluate : $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$

OR

(b) Find : $\int \frac{2x+1}{(x+1)^2(x-1)} dx$

25. Find : $\int \frac{2x}{(x^2+1)(x^2+3)} dx$

26. (a) Evaluate : $\int_{-6}^{6} |x+2|\ dx$

OR

(b) Find : $\int \left(\frac{4-x}{x^5}\right) e^x dx$

27. (a) Find : $\int \frac{dx}{\cos x\sqrt{\cos 2x}}$

OR

(b) Find : $\int \frac{5x-3}{\sqrt{1+4x-2x^2}} dx$

Section D: Long Answer Type Questions (5 Marks)

28. (a) Evaluate : $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$

OR

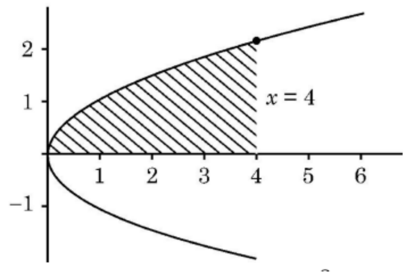
(b) Evaluate : $\int_0^{\frac{\pi}{2}} \sin 2x \tan^{-1}(\sin x)dx$

8. APPLICATIONS OF INTEGRALS

March, 2025

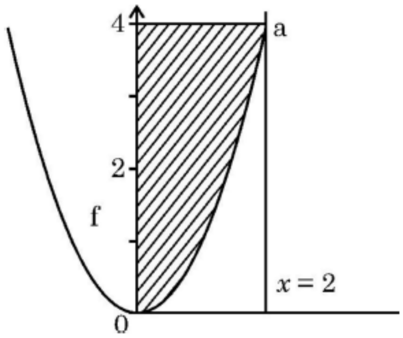
Multiple Choice Questions (1 Mark)

1. The area of the shaded region bounded by the curves $y^2 = x, x = 4$ and the x-axis is given by:



(A) $\int_0^4 x dx$
(B) $\int_0^2 y^2 dy$
(C) $2\int_0^4 \sqrt{x} dx$
(D) $\int_0^4 \sqrt{x} dx$

2. The area of the shaded region (figure) represented by the curves $y = x^2,\ 0 \le x \le 2$ and y-axis is given by:



(A) $\int_0^2 x^2\, dx$
(B) $\int_0^2 \sqrt{y}\, dy$
(C) $\int_0^4 x^2\, dx$
(D) $\int_0^4 \sqrt{y}\, dy$

3. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines x = 0 and x = 4 and x-axis is :

(A) $\frac{16}{9}$ sq. units (B) $\frac{32}{9}$ sq. units
(C) $\frac{16}{3}$ sq. units (D) $\frac{32}{3}$ sq. units

4. The area of the region enclosed between the curve $y = x|x|$, x-axis, x = – 2 and x = 2 is :

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$
(C) 0 (D) 8

5. The area bounded by the parabola $x^2 = y$ and the line $y = 1$ is :

(A) $\frac{2}{3}$ sq unit (B) $\frac{1}{3}$ sq unit
(C) $\frac{4}{3}$ sq units (D) 2 sq units

6. The area of the region bounded by the lines $y = x + 1, x = 1, x = 3$ and x-axis is

(A) 6 sq units (B) 8 sq units
(C) 7.5 sq units (D) 2 sq units

7. The area enclosed by the curve $y = \sqrt{4 - x^2}$ and the coordinate axes in the first quadrant is :

(A) 2π sq. units (B) π sq. units
(C) $\frac{\pi}{2}$ sq. units (D) 4π sq. units

Very Short Answer Questions (2 Marks)

8. Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.

Short Answer & Long Answer Questions (3 - 5 Marks)

9. Sketch the graph of $y = |\ x + 3\ |$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

10. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the x-axis and the ordinates $x = -2$ and $x = 2$.

11. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9,\ x = 0$ and $y = x^2$.

12. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

13. Using integration, find the area of the region $\{(x, y) : 0 \le y \le x^2,\ 0 \le y \le x,\ 0 \le x \le 3\}$.

14. The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x =$ a. Find the value of a.

15. Using integration, find the area of the region bounded between the lines $x = -2, x = 2$ and the circle $x^2 + y^2 = 16$.

March, 2024

Section A: Multiple Choice Questions (1 Mark)

1. Area of the region bounded by curve $y^2 = 4x$ and the X-axis between $x = 0$ and $x = 1$ is :

(A) $\frac{2}{3}$ (B) $\frac{8}{3}$
(C) 3 (D) $\frac{4}{3}$

2. The area (in sq. units) of the region bounded by the curve y = x, x-axis, x = 0 and x = 2 is :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}\log 2$
(C) 2 (D) 4

Section D: Long Answer Type Questions (5 Marks)

3. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines x = – 2 and x = 2.

4. If A_1 denotes the area of region bounded by $y^2 = 4x, x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x, x = 4$, find $A_1 : A_2$.

5. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$.

6. (a) Sketch the graph of $y = x|x|$ and hence find the area bounded by this curve, X-axis and the ordinates $x = -2$ and $x = 2$, using integration.

OR

(b) Using integration, find the area bounded by the ellipse $9x^2 + 25y^2 = 225$, the lines $x = -2$, $x = 2$, and the X-axis.

7. Find the area of the region bounded by the curve $4x^2 + y^2 = 36$ using integration.

8. Using integration, find the area of the region bounded by the curve $y = x^2, x = -1, x = 1$ and the x-axis.

9. (a) Using integration, find the area of the region bounded by the curve $y = \sqrt{4 - x^2}$, the lines $x = -\sqrt{2}$ and $x = \sqrt{3}$ and the x-axis.

OR

(b) Using integration, evaluate the area of the region bounded by the curve $y = x^2$, the lines y = 1 and y = 3 and the y-axis.

9. DIFFERENTIAL EQUATIONS

March, 2025

Multiple Choice Questions (1 Mark)

1. The integrating factor of differential equation $(x+2y^3)\frac{dy}{dx} = 2y$ is

(A) $e^{\frac{y^2}{2}}$ (B) $\frac{1}{\sqrt{y}}$
(C) $\frac{1}{y^2}$ (D) $e^{-\frac{1}{y^2}}$

2. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p-q)$ is

(A) 0 (B) 1
(C) 2 (D) 3

3. The order and degree of the differential equation
$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ are :

(A) order 2, degree 2
(B) order 2, degree 1
(C) order 2, degree not defined
(D) order 1, degree not defined

4. The integrating factor of the differential equation
$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ is :

(A) $e^{-\frac{1}{\sqrt{x}}}$ (B) $e^{\frac{2}{\sqrt{x}}}$
(C) $e^{2\sqrt{x}}$ (D) $e^{-2\sqrt{x}}$

5. The sum of the order and degree of the differential equation
$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ is :

(A) 2 (B) $\frac{5}{2}$
(C) 3 (D) 4

6. The general solution of the differential equation $\frac{dy}{dx} = 2x \cdot e^{x^2+y}$ is :

(A) $e^{x^2+y} = C$ (B) $e^{x^2} + e^{-y} = C$
(C) $e^{x^2} = e^y + C$ (D) $e^{x^2-y} = C$

7. If 'm' and 'n' are the degree and order respectively of the differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \frac{d^2y}{dx^2}$, then the value of $(m+n)$ is :

(A) 4 (B) 3
(C) 2 (D) 5

8. The integrating factor for solving the differential equation $x \cdot \frac{dy}{dx} - y = 2x^2$ is

(A) x (B) $\frac{1}{x}$
(C) e^{-x} (D) $-\log x$

9. The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is :

(A) $\tan^{-1}(y-x) = C$
(B) $\tan^{-1} y + \tan^{-1} x = C$
(C) $\tan^{-1}\frac{y}{2} - \tan^{-1}\frac{x}{2} = C$
(D) $\tan^{-1} y = \tan^{-1} x + C$

10. The integrating factor for solving the differential equation $\tan x\frac{dy}{dx} + y = x^2, (x \neq 0)$ is :

(A) e^x (B) $\tan x$
(C) $\sin x$ (D) $\frac{1}{\sin}x$

Assertion - Reason Questions (1 Mark)

11. Assertion (A) : $x^2dy = (2xy+y^2)dx$ is a homogeneous differential equation.

Reason (R) : A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true and Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is true.

Short Answer & Long Answer Questions (3 - 5 Marks)

12. (a) Solve the differential equation $2(y+3) - xy\frac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation: $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2$.

13. Find the particular solution of the differential equation

$\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\ dy = 0$

given that $y = \frac{\pi}{4}$, when x = 1.

14. Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

15. (a) Find the general solution of the differential equation

$(2x^2 + y)dx = xdy$.

OR

(b) For the differential equation $\frac{dy}{dx} - \frac{y}{x} + \csc\left(\frac{y}{x}\right) = 0$, find the particular solution, given that $y = 0$ when $x = 1$.

16. (a) Find the particular solution of the differential equation, $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that $y = 1$ when $x = 0$.

OR

(b) Solve the differential equation : $2xy\frac{dy}{dx} = x^2 + 3y^2$.

17. (a) Solve the differential equation $y + x\frac{dy}{dx} = x - y\frac{dy}{dx}$.

OR

(b) Find the particular solution of the differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given that $y = 1$ when $x = \frac{\pi}{2}$.

Case Study Based Questions (4 Marks)

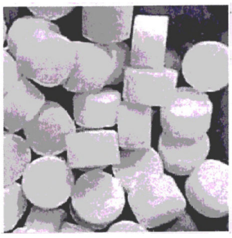
18. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$, where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, T(t) given that T(0) = 85°C.

(ii) How long will it take for the processor's temperature to reach 40°C ? Given that k = 0.03, $\log_e 4 = 1.3863$.

19. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation.

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm.

(iii) (a) If it is given that r = 3 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm.

OR

(iii) (b) If it is given that r = 1 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm.

March, 2024

Section A: Multiple Choice Questions (1 Mark)

1. The integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is :

(A) $\frac{1}{x^2-1}$ (B) $\frac{1}{\sqrt{x^2-1}}$
(C) $\frac{1}{1-x^2}$ (D) $\frac{1}{\sqrt{1-x^2}}$

2. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are :

(A) 1, 2 (B) 2, 3
(C) 2, 1 (D) 2, 6

3. The differential equation $\frac{dy}{dx} = F(x, y)$ will not be a homogeneous differential equation, if F(x, y) is :

(A) $\cos x - \sin\left(\frac{y}{x}\right)$ (B) $\frac{y}{x}$
(C) $\frac{x^2+y^2}{xy}$ (D) $\cos^2\left(\frac{x}{y}\right)$

4. The degree of the differential equation $(y'')^2 + (y')^3 = x\sin(y')$ is :

(A) 1 (B) 2
(C) 3 (D) not defined

5. $x\log x\frac{dy}{dx} + y = 2\log x$ is an example of a :

(A) variable separable differential equation.
(B) homogeneous differential equation.
(C) first order linear differential equation.
(D) differential equation whose degree is not defined.

6. The general solution of the differential equation $x\ dy + y\ dx = 0$ is :

(A) $xy = c$ (B) $x + y = c$
(C) $x^2 + y^2 = c^2$ (D) $\log y = \log x + c$

7. The integrating factor of the differential equation $(x + 2y^2)\frac{dy}{dx} = y$ $(y > 0)$ is :

(A) $\frac{1}{x}$ (B) x
(C) y (D) $\frac{1}{y}$

8. The order of the differential equation $\frac{d^4y}{dx^4} - \sin\left(\frac{d^2y}{dx^2}\right) = 5$ is :

(A) 4 (B) 3
(C) 2 (D) not defined

9. The solution of the differential equation $\frac{dy}{dx} = 1 - x + y - xy$ is :

(A) $\log|1+y| = x - \frac{x^2}{2} + c$
(B) $\log|1+y| = -x + \frac{x^2}{2} + c$

(C) $e^y = x - \frac{x^2}{2} + c$
(D) $e^{1+y} = -x + \frac{x^2}{2} + c$

10. The degree of the differential equation $x\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^5 = 0$ is :

(A) 2 (B) 3
(C) 4 (D) 5

11. The integrating factor of the differential equation $\frac{dy}{dx} + y\tan x = 2x + x^2\tan x$ is :

(A) $e^{\sec x}$ (B) $\sec x + \tan x$
(C) $\sec x$ (D) $\cos x$

12. The number of arbitrary constants in the general solution of the differential equation $\frac{dy}{dx} + y = 0$ is :

(A) 0 (B) 1
(C) 2 (D) 3

Section C: Short Answer Type Questions (3 Marks)

13. Find the particular solution of the differential equation given by $x^2\frac{dy}{dx} - xy = x^2\cos^2\left(\frac{y}{2x}\right)$, given that when x = 1, $y = \frac{\pi}{2}$.

14. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2\frac{dy}{dx} = 0; y = 2$, when x = 1.

OR

(b) Find the general solution of the differential equation :
$ydx = (x + 2y^2)dy$

15. (a) Find the particular solution of the differential equation $\frac{dy}{dx} = y\cot 2x$, given that $y(\frac{\pi}{4}) = 2$.

OR

(b) Find the particular solution of the differential equation $\left(xe^{\frac{y}{x}} + y\right)dx = x\ dy$, given that $y = 1$ when $x = 1$.

16. (a) Find the particular solution of the differential equation
$\frac{dy}{dx} - 2xy = 3x^2e^{x^2}; y(0) = 5$.

OR

(b) Solve the following differential equation :
$x^2dy + y(x + y)dx = 0$

17. (a) Find the particular solution of the differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$, given that $y(1) = 0$.

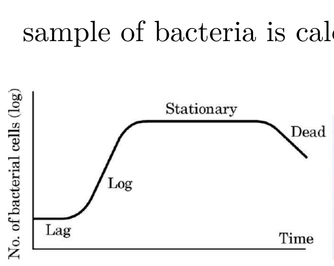
OR

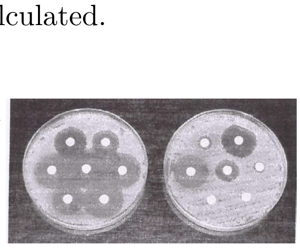
(b) Solve the differential equation $\frac{dy}{dx} + 2y\tan x = \sin x$, given that $y = 0$, when $x = \frac{\pi}{3}$.

18. Find the general solution of the differential equation
$ydx - xdy + (x\log x)dx = 0$.

Section E: Case Study Based Questions (4 Marks)

19. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :
$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time ‘t’.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of ‘t’.

(ii) If population of bacteria is 1000 at t = 0, and 2000 at t = 1, find the value of k.

10. VECTOR ALGEBRA

March, 2025

Multiple Choice Questions (1 Mark)

1. If vector $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and vector $\vec{b} = \hat{\imath} - \hat{\jmath} + \hat{k}$, then which of the following is correct ?

(A) $\vec{a} \parallel \vec{b}$ (B) $\vec{a} \perp \vec{b}$
(C) $|\vec{b}| > |\vec{a}|$ (D) $|\vec{a}| = |\vec{b}|$

2. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$ (D) $\frac{\pi}{2}$

3. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

(A) $\frac{\vec{a}\cdot\vec{b}}{|}\vec{b}|^2\ \vec{b}$ (B) $\frac{\vec{a}\cdot\vec{b}}{|}\vec{b}|$
(C) $\frac{\vec{a}\cdot\vec{b}}{|}\vec{a}|$ (D) $\frac{\vec{a}\cdot\vec{b}}{|}\vec{a}|^2\ \vec{b}$

4. Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p} + \vec{q})$ will be a unit vector for what value of α?

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $\frac{\pi}{2}$ (D) $2\frac{\pi}{3}$

5. If the sides AB and AC of Δ ABC are represented by vectors $\hat{\jmath} + \hat{k}$ and $3\hat{\imath} - \hat{\jmath} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(A) $2\sqrt{2}$ units (B) $\sqrt{18}$ units
(C) $\frac{\sqrt{34}}{2}$ units (D) $\frac{\sqrt{48}}{2}$ units

6. Let $\vec{a}$ be a position vector whose tip is the point (2, – 3). If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are (– 4, 5), then the coordinates of B are :

(A) (– 2, – 2) (B) (2, – 2)
(C) (– 2, 2) (D) (2, 2)

7. The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $\left(\vec{a} - \vec{b}\right) \cdot \left(\vec{a} + \vec{b}\right) = 512$ and $|\vec{a}| = 3\ |\vec{b}|$, are :

(A) 48 and 16 (B) 3 and 1
(C) 24 and 8 (D) 6 and 2

8. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{\imath} + 15\hat{\jmath} + 6\hat{k}$ and the other is along the vector $2\hat{\imath} + 10\hat{\jmath} + \lambda\hat{k}$, then the value of λ is :

(A) 6 (B) 1
(C) $\frac{1}{4}$ (D) 4

9. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors
(B) parallel to each other
(C) unit vectors
(D) collinear vectors

10. If $|\vec{a}| = 1, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 2$, then the value of $|\vec{a} + \vec{b}|$ is :

(A) 9 (B) 3
(C) -3 (D) 2

11. Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$. The angle between the two vectors is :

(A) 30° (B) 60°
(C) 45° (D) 90°

12. The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$ is (are) :

(A) one (B) two
(C) three (D) infinite

13. The value of x for which $x\left(\hat{\imath}+\hat{\jmath}+\hat{k}\right)$ is a unit vector, is :

(A) 1 (B) $\pm\sqrt{3}$
(C) $\pm\frac{1}{\sqrt{3}}$ (D) ± 3

Assertion - Reason Questions (1 Mark)

14. Assertion (A) : The vectors $\vec{a}=4\hat{\imath}+\hat{\jmath}-\hat{k}$ and $\vec{b}=-2\hat{\imath}+3\hat{\jmath}-5\hat{k}$ are mutually perpendicular vectors.

Reason (R) : Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a}\cdot\vec{b}=0$.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

Very Short Answer Questions (2 Marks)

15. The diagonals of a parallelogram are given by $\vec{a}=2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\vec{b}=\hat{\imath}+3\hat{\jmath}-\hat{k}$. Find the area of the parallelogram.

16. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a}=3\hat{\imath}+\hat{\jmath}+2\hat{k}$ and $\vec{b}=2\hat{\imath}-2\hat{\jmath}+4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, –1, 0) respectively.

17. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that QR = $\frac{3}{2}$ QP.

18. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha}=(x-2)\vec{a}+\vec{b}$ and $\vec{\beta}=(3+2x)\vec{a}-2\vec{b}$ are collinear.

19. (a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{\imath}-2\hat{\jmath}+\hat{k}$ and $4\hat{\imath}+3\hat{\jmath}-2\hat{k}$.

OR

(b) Let $\vec{a},\vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a}\cdot\vec{b}=\vec{a}\cdot\vec{c}$ and $\vec{a}\times\vec{b}=\vec{a}\times\vec{c},\ \vec{a}\neq 0$. Show that $\vec{b}=\vec{c}$.

20. (a) Find the value of λ, if the points $(-1,-1,2),(2,8,\lambda)$ and $(3,11,6)$ are collinear.

OR

(b) $\vec{a}$ and $\vec{b}$ are two co-initial vectors forming the adjacent sides of a parallelogram such that $|\vec{a}|=10, |\vec{b}|=2$ and $\vec{a}\cdot\vec{b}=12$. Find the area of the parallelogram.

21. (a) If $|\vec{a}+\vec{b}|=60$, $|\vec{a}-\vec{b}|=40$ and $|\vec{b}|=46$, then find $|\vec{a}|$.

OR

(b) Using vectors, find the value of K such that the points (K, –11, 2), (0, –2, 2) and (2, 4, 2) are collinear.

22. (a) If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=2, |\vec{b}|=3$ and $|\vec{c}|=4$, then find the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$.

OR

(b) Let $\vec{a}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$, $\vec{b}=4\hat{\imath}-3\hat{\jmath}+9\hat{k}$ and $\vec{c}=3\hat{\imath}-2\hat{\jmath}+6\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\cdot\vec{d}=39$.

Short Answer & Long Answer Questions (3 - 5 Marks)

23. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}\ |\vec{a}\times\vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\hat{\imath}+3\hat{\jmath}-\hat{k}$.

24. (a) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ such that $|\vec{a}|=3,\ \ |\vec{b}|=5,\ \ |\vec{c}|=7$, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}\ |\vec{a}-\vec{b}|=\sin\left(\frac{\theta}{2}\right)$.

25. (a) The scalar product of the vector $\vec{a}=\hat{\imath}-\hat{\jmath}+2\hat{k}$ with a unit vector along sum of vectors $\vec{b}=2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\vec{c}=\lambda\hat{\imath}-2\hat{\jmath}-3\hat{k}$ is equal to 1. Find the value of λ.

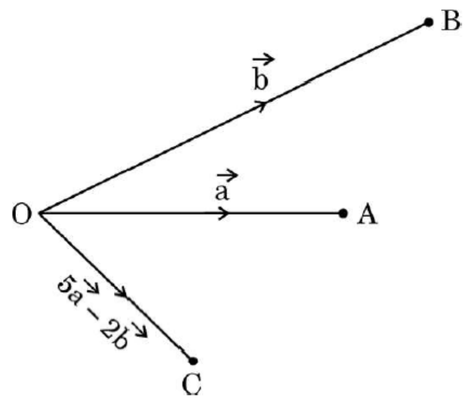
26. If $\hat{a}, \hat{b}$ and $\hat{c}$ are unit vectors such that $\hat{a}\cdot\hat{b}=\hat{a}\cdot\hat{c}=0$ and the angle between $\hat{b}$ and $\hat{c}$ is $\frac{\pi}{6}$, then prove that $\hat{a}=\pm 2\left(\hat{b}\times\hat{c}\right)$.

27. If $\vec{a}=\hat{\imath}+2\hat{\jmath}+\hat{k}$, $\vec{b}=2\hat{\imath}+\hat{\jmath}$ and $\vec{c}=3\hat{\imath}-4\hat{\jmath}-5\hat{k}$, then find a unit vector perpendicular to both the vectors $\left(\vec{a}-\vec{b}\right)$ and $\left(\vec{c}-\vec{b}\right)$.

28. Using vectors, find the area of triangle ABC with vertices A(4, 3, 3), B(5, 5, 6) and C(4, 7, 6).

Case Study Based Questions (4 Marks)

29. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA}=\vec{a}$, $\overrightarrow{OB}=\vec{b}$ and $\overrightarrow{OC}=5\vec{a}-2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors.

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

(iii) (a) If $\vec{a}\cdot\vec{b}=1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a}\times\vec{b}|$.

OR

(iii) (b) If $\vec{a}=2\hat{\imath}-\hat{\jmath}+4\hat{k}$ and $\vec{b}=\hat{\jmath}-\hat{k}$, then find a unit vector perpendicular to $\left(\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-\vec{b}\right)$.

March, 2024

Section A: Multiple Choice Questions (1 Mark)

1. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is equal to :

(A) $\pm\frac{3}{5}$ (B) $\pm\frac{3}{4}$
(C) $\pm\frac{4}{5}$ (D) $\pm\frac{4}{3}$

2. The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(A) $\hat{\imath} - \hat{\jmath} + 2\hat{k}$ (B) $\hat{\imath} + \hat{\jmath} + 2\hat{k}$
(C) $-\hat{\imath} - \hat{\jmath} - 2\hat{k}$ (D) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$

3. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}|\ |\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}|\ |\vec{b}|$
(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}|\ |\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}|\ |\vec{b}|$

4. The unit vector perpendicular to both vectors $\hat{\imath} + \hat{k}$ and $\hat{\imath} - \hat{k}$ is :

(A) $2\hat{\jmath}$ (B) $\hat{\jmath}$
(C) $\frac{\hat{\imath} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{\imath} + \hat{k}}{\sqrt{2}}$

5. If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel
(B) parallel vectors
(C) perpendicular vectors
(D) unit vectors

6. If $|\vec{a}| = 2$ and $-3 \leq k \leq 2$, then $|k\vec{a}| \in$:

(A) [– 6, 4] (B) [0, 4]
(C) [4, 6] (D) [0, 6]

7. If $\vec{a}$ and $\vec{b}$ are two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = \sqrt{3}$, then the angle between $2\vec{a}$ and $-\vec{b}$ is :

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$
(C) $5\frac{\pi}{6}$ (D) $11\frac{\pi}{6}$

8. The vectors $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} - 3\hat{\jmath} - 5\hat{k}$ and $\vec{c} = -3\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$ represents the sides of

(A) an equilateral triangle
(B) an obtuse-angled triangle
(C) an isosceles triangle
(D) a right-angled triangle

9. Let $\vec{a}$ be any vector such that $|\vec{a}| = a$. The value of $|\vec{a} \times \hat{\imath}|^2 + |\vec{a} \times \hat{\jmath}|^2 + |\vec{a} \times \hat{k}|^2$ is :

(A) a^2 (B) $2a^2$
(C) $3a^2$ (D) 0

10. The position vectors of points P and Q are $\vec{p}$ and $\vec{q}$ respectively. The point R divides line segment PQ in the ratio 3 : 1 and S is the mid-point of line segment PR. The position vector of S is :

(A) $\frac{\vec{p}+3\vec{q}}{4}$ (B) $\frac{\vec{p}+3\vec{q}}{8}$
(C) $\frac{5\vec{p}+3\vec{q}}{4}$ (D) $\frac{5\vec{p}+3\vec{q}}{8}$

11. Let the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}| = \sqrt{3}$ and $|\vec{b}| = \frac{2}{\sqrt{3}}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$ (D) $\frac{\pi}{2}$

12. If $\vec{a} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{c} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, then the projection of $\left(\vec{c} - \vec{b}\right)$ along $\vec{a}$ is :

(A) 15 (B) 5
(C) $\frac{2}{3}$ (D) 1

13. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\left(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right)$ is equal to :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$
(C) $-\frac{1}{2}$ (D) $-\frac{3}{2}$

14. What is the value of $\frac{\text{projection of } \vec{a} \text{ on } \vec{b}}{\text{projection of } \vec{b} \text{ on } \vec{a}}$ for vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} - 6\hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$?

(A) $\frac{3}{7}$ (B) $\frac{7}{3}$
(C) $\frac{4}{3}$ (D) $\frac{4}{7}$

15. If $\vec{a}$ and $\vec{b}$ are two vectors such that $\vec{a} \cdot \vec{b} > 0$ and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$
(C) $2\frac{\pi}{3}$ (D) $3\frac{\pi}{4}$

Section A: Assertion-Reason (1 Mark)

16. Assertion (A) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

17. Assertion (A) : Projection of $\vec{a}$ on $\vec{b}$ is same as projection of $\vec{b}$ on $\vec{a}$.

Reason (R) : Angle between $\vec{a}$ and $\vec{b}$ is same as angle between $\vec{b}$ and $\vec{a}$ numerically.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

18. Assertion (A) : The vectors
$\vec{a} = 6\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$
$\vec{b} = 10\hat{\imath} - 2\hat{\jmath} - 6\hat{k}$
$\vec{c} = 4\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$
represent the sides of a right angled triangle.

Reason (R) : Three non-zero vectors of which none of two are collinear forms a triangle if their resultant is zero vector or sum of any two vectors is equal to the third.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B: Very Short Answer Type Questions (2 Marks)

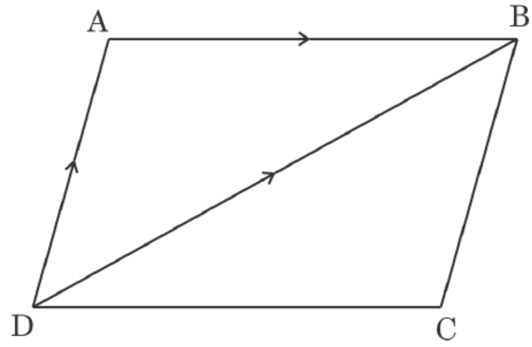
19. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $-\hat{\imath} + \hat{\jmath} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

20. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors.
Prove that $|\vec{a} \times \vec{b}| \leq |\vec{a}|\ |\vec{b}|$.
State the condition under which equality holds, i.e., $|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}|$.

Section C: Short Answer Type Questions (3 Marks)

21. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $\left(\vec{a}+\vec{b}\right) \perp \vec{a}$ and $\left(2\vec{a}+\vec{b}\right) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\ |\vec{a}|$.

22. In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.



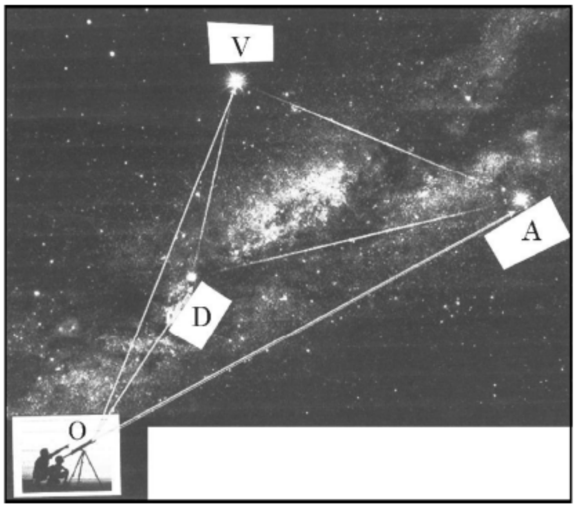
23. The position vectors of vertices of Δ ABC are $A(2\hat{\imath} - \hat{\jmath} + \hat{k})$, $B(\hat{\imath} - 3\hat{\jmath} - 5\hat{k})$ and $C(3\hat{\imath} - 4\hat{\jmath} - 4\hat{k})$. Find all the angles of Δ ABC.

24. Find a vector of magnitude 4 units perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\hat{\imath} + \hat{\jmath} - \hat{k}$ and hence verify your answer.

25. If the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ represent the three sides of a triangle, then show that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$.

Section E: Case Study Based Questions (4 Marks)

26. An instructor at the astronomical centre shows three among the brightest stars in a particular constellation. Assume that the telescope is located at O(0, 0, 0) and the three stars have their locations at the points D, A and V having position vectors $2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $7\hat{\imath} + 5\hat{\jmath} + 8\hat{k}$ and $-3\hat{\imath} + 7\hat{\jmath} + 11\hat{k}$ respectively.



Based on the above information, answer the following questions :

(i) How far is the star V from star A ?

(ii) Find a unit vector in the direction of $\overrightarrow{DA}$.

(iii) Find the measure of $\angle$ VDA.

OR

(iii) What is the projection of vector $\overrightarrow{DV}$ on vector $\overrightarrow{DA}$?

27. A cricket match is organised between two clubs P and Q for which a team from each club is chosen. Remaining players of club P and club Q are respectively sitting along the lines AB and CD, where the points are A(3, 4, 0), B(5, 3, 3), C(6, – 4, 1) and D(13, – 5, – 4). Based on the above, answer the following questions :

(i) Write the direction ratios of vector $\overrightarrow{AB}$.

(ii) Write a unit vector in the direction of $\overrightarrow{CD}$.

(iii) (a) Find the angle between vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

OR

(iii) (b) Write a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

11. THREE DIMENSIONAL GEOMETRY

March 2025

Multiple Choice Questions (1 Mark)

1. If a line makes angles of $\frac{3\pi}{4}$, $\frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is
(A) $\frac{-\pi}{3}$ only
(B) $\frac{\pi}{3}$ only
(C) $\frac{\pi}{6}$
(D) $\pm\frac{\pi}{3}$

2. The equation of a line parallel to the vector $3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is:
(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$
(B) $x = 3t + 4, y = t + 3, z = 2t + 7$
(C) $x = 3t + 4, y = t - 3, z = 2t + 7$
(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

3. The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point?
(A) $(1, -5, 6)$
(B) $(1, 5, 6)$
(C) $(1, -5, -6)$
(D) $(-1, -5, 6)$

4. If P is a point on the line segment joining $(3, 6, -1)$ and $(6, 2, -2)$ and y-coordinate of P is 4, then its z-coordinate is :
(A) $-\frac{3}{2}$
(B) 0
(C) 1
(D) $\frac{3}{2}$

Assertion - Reason Questions (1 Mark)

5. Assertion (A) : $\langle\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{5}}\rangle$ cannot be the direction cosines of a line.
Reason (R) : If l, m, n are the direction cosines of a line, then $l^2 + m^2 + n^2 = 1$.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Very Short Answer Questions (2 Marks)

6. A man needs to hang two lanterns on a straight wire whose end points have coordinates A $(4, 1, -2)$ and B $(6, 2, -3)$. Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.

7. Find the angle between the lines
$\vec{r} = (3 + 2\lambda)\hat{\imath} - (2 - 2\lambda)\hat{\jmath} + (6 + 2\lambda)\hat{k}$ and
$\vec{r} = \left(2\hat{\jmath} - 5\hat{k}\right) + \mu\left(6\hat{\imath} + 3\hat{\jmath} + 2\hat{k}\right)$.

8. Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.

9. If the equations of a line are $\frac{x-2}{2} = \frac{2y-5}{-3}, z = -1$, then find the direction ratios of the line and also a point on the line.

Short Answer & Long Answer Questions (3 - 5 Marks)

10. (a) Verify that lines given by $\vec{r} = (1 - \lambda)\hat{\imath} + (\lambda - 2)\hat{\jmath} + (3 - 2\lambda)\hat{k}$ and $\vec{r} = (\mu + 1)\hat{\imath} + (2\mu - 1)\hat{\jmath} - (2\mu + 1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{\imath} + 8\hat{\jmath}$, $\vec{W} = 6\hat{\imath} + 12\hat{\jmath}$ and $\vec{F} = 12\hat{\imath} + 18\hat{\jmath}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

11. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point Q

$(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q.

12. (a) Find the shortest distance between the lines: $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ and $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$.

OR

(b) Find the image A' of the point $A(2, 1, 2)$ in the line $l : (r) = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k} + \lambda\left(\hat{\imath} - \hat{\jmath} - \hat{k}\right)$. Also, find the equation of line joining AA'. Find the foot of perpendicular from point A on the line l.

13. Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

14. (b) Find the equation of a line in vector and cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and $(r) = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k} + \mu\left(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}\right)$.

15. (a) Find the foot of the perpendicular drawn from the point $(1, 1, 4)$ on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

OR

(b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point $(-1, -1, 2)$.

16. (b) Find the shortest distance between the lines : $(r) = \left(2\hat{\imath} - \hat{\jmath} + 3\hat{k}\right) + \lambda\left(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}\right)$ $(r) = \left(\hat{\imath} + 4\hat{k}\right) + \mu\left(3\hat{\imath} - 6\hat{\jmath} + 9\hat{k}\right)$.

17. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P $(1, 2, 3)$.

OR

(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

18. (a) Find the shortest distance between the lines l_1 and l_2 given by : $l_1 : (r) = \hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda\left(4\hat{\imath} + 6\hat{\jmath} + 12\hat{k}\right)$ and $l_2 : (r) = 3\hat{\imath} + 3\hat{\jmath} - 5\hat{k} + \mu\left(6\hat{\imath} + 9\hat{\jmath} + 18\hat{k}\right)$

OR

(b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = \frac{z}{1}$ intersect. Also, find their point of intersection.

19. (a) Find the shortest distance between the lines given by $(\mathrm{r}) = \left(4\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} - 3\hat{k}\right)$ and $(\mathrm{r}) = \left(2\hat{\imath} + \hat{\jmath} - \hat{k}\right) + \mu\left(3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}\right)$

OR

(b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line $(\mathrm{r}) = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda\left(2\hat{\imath} + 3\hat{\jmath} - \hat{k}\right)$.

20. (a) Write the nature of the lines $\frac{x-1}{4} = \frac{y-2}{6} = \frac{z-3}{8}$ and $\frac{x-2}{2} = \frac{y-4}{3} = \frac{z-5}{4}$. Also, find the shortest distance between them.

OR

(b) Show that the lines $(r) = \left(2\hat{\jmath} - 3\hat{k}\right) + \lambda\left(\hat{\imath} + 2\hat{\jmath} + 3\hat{k}\right)$ and $(r) = \left(2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}\right) + \mu\left(2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}\right)$ intersect. Also find their point of intersection.

Case Study Based Questions (4 Marks)

21. An engineer is designing a new metro rail network in a city. Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions : (i) Find whether the two metro tracks are parallel. **1** (ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. **1** (iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point $(3, 2, 1)$. Determine the equation of the pedestrian walkway. **2**

OR

(iii) (b) Find the shortest distance between Line A and Line B. **2**

March 2024

Section A: Multiple Choice Questions (1 Mark)

1. If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is :
(A) ± 1
(B) $\pm\sqrt{3}$
(C) ± 3
(D) $\pm\frac{1}{3}$

2. The distance of point P(a, b, c) from y-axis is :
(A) b
(B) b^2
(C) $\sqrt{a^2+c^2}$
(D) a^2+c^2

3. The coordinates of the foot of the perpendicular drawn from the point $(0, 1, 2)$ on the x-axis are given by :
(A) $(1, 0, 0)$
(B) $(2, 0, 0)$
(C) $\left(\sqrt{5}, 0, 0\right)$
(D) $(0, 0, 0)$

4. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :
(A) 2, -1, 6
(B) 2, 1, 6
(C) 2, 1, 3
(D) 2, -1, 3

5. If a line makes an angle of $30°$ with the positive direction of x-axis, $120°$ with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :
(A) $90°$
(B) $120°$
(C) $60°$
(D) $0°$

6. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is **not** true ?
(A) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
(B) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$
(D) $\cos\alpha + \cos\beta + \cos\gamma = 1$

7. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :
(A) 0
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{2}$
(D) π

8. The vector equation of a line passing through the point $(1, -1, 0)$ and parallel to Y-axis is :
(A) $(r) = \hat{\imath} - \hat{\jmath} + \lambda(\hat{\imath} - \hat{\jmath})$
(B) $(r) = \hat{\imath} - \hat{\jmath} + \lambda\hat{\jmath}$
(C) $(r) = \hat{\imath} - \hat{\jmath} + \lambda\hat{k}$
(D) $(r) = \lambda\hat{\jmath}$

9. The lines $\frac{1-x}{2} = \frac{y-1}{3} = \frac{z}{1}$ and $\frac{2x-3}{2p} = \frac{y}{-1} = \frac{z-4}{7}$ are perpendicular to each other for p equal to :
(A) $-\frac{1}{2}$
(B) $\frac{1}{2}$
(C) 2
(D) 3

10. The angle which the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{0}$ makes with the positive direction of Y-axis is :
(A) $\frac{5\pi}{6}$
(B) $\frac{3\pi}{4}$
(C) $\frac{5\pi}{4}$
(D) $\frac{7\pi}{4}$

11. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line : $(r) = (2+\lambda)\hat{\imath} + \lambda\hat{\jmath} + (2\lambda - 1)\hat{k}$ is
(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$
(B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$
(D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$

12. The angle between the lines $\frac{x+1}{2} = \frac{2-y}{-5} = \frac{z}{4}$ and $\frac{x-3}{1} = \frac{y-7}{2} = \frac{5-z}{3}$ is :
(A) $\frac{\pi}{4}$
(B) $\frac{\pi}{2}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{6}$

13. The Cartesian equations of a line are given as $6x - 2 = 3y + 1 = 2z - 2$ The direction ratios of the line are :
(A) 2, -1, 3
(B) 1, -2, -3

(C) 1, 2, 3
(D) 3, 1, 2

14. The direction ratios of the line $\frac{x-1}{3} = \frac{2-y}{1} = \frac{3z}{2}$ are :
(A) 3, 1, 2
(B) 4, 3, 2
(C) 9, −3, 2
(D) 9, 3, 2

15. The Cartesian equation of the line passing through the point $(1, -3, 2)$ and parallel to the line $(r) = 2\hat{\imath} - \hat{k} + \lambda\left(\hat{\imath} + \hat{\jmath} + 2\hat{k}\right)$ is :
(A) $\frac{x-1}{2} = \frac{y+3}{0} = \frac{z-2}{-1}$
(B) $\frac{x+1}{1} = \frac{y-3}{1} = \frac{z+2}{2}$
(C) $\frac{x+1}{2} = \frac{y-3}{0} = \frac{z+2}{-1}$
(D) $\frac{x-1}{1} = \frac{y+3}{1} = \frac{2-z}{-2}$

Section A: Assertion-Reason (1 Mark)

16. (Assertion-Reason)
Assertion (A) : A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.
Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section B: Very Short Answer Type Questions (2 Marks)

17. Find the vector equation of the line passing through the point $(2, 3, -5)$ and making equal angles with the co-ordinate axes.

18. Find the angle between the lines $\frac{5-x}{-7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$.

Section D: Long Answer Type Questions (5 Marks)

19. The image of point $P(x, y, z)$ with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is P' $(1, 0, 7)$. Find the coordinates of point P.

20. Find the equation of the line which bisects the line segment joining points $A(2, 3, 4)$ and B $(4, 5, 8)$ and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

21. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as $A(-1, 2, 1)$ and $B(1, -2, 5)$. If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

22. (a) Find the distance between the line $\frac{x}{2} = \frac{2y-6}{4} = \frac{1-z}{-1}$ and another line parallel to it passing through the point $(4, 0, -5)$.

OR

(b) If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-7}$ are perpendicular to each other, find the value of k and hence write the vector equation of a line perpendicular to these two lines and passing through the point $(3, -4, 7)$.

23. (a) Find the co-ordinates of the foot of the perpendicular drawn from the point $(2, 3, -8)$ to the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$. Also, find the perpendicular distance of the given point from the line.

OR

(b) Find the shortest distance between the lines L_1 & L_2 given below : L_1 : The line passing through $(2, -1, 1)$ and parallel to $\frac{x}{1} = \frac{y}{1} = \frac{z}{3}$ $L_2 : (r) = \hat{\imath} + (2\mu + 1)\hat{\jmath} - (\mu + 2)\hat{k}$.

24. (a) Write the vector equations of the following lines and hence find the shortest distance between them : $\frac{x+1}{2} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

OR

(b) Find the length and the coordinates of the foot of the perpendicular drawn from the point P$(5, 9, 3)$ to the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Also, find the coordinates of the image of the point P in the given line.

25. (a) Find the shortest distance between the lines $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

OR

(b) Find the point of intersection of the lines $(r) = \hat{\imath} - \hat{\jmath} + 6\hat{k} + \lambda\left(3\hat{\imath} - \hat{k}\right)$, and $(r) = -3\hat{\jmath} + 3\hat{k} + \mu\left(\hat{\imath} + 2\hat{\jmath} - \hat{k}\right)$. Also, find the vector equation of the line passing through the point of intersection of the given lines and perpendicular to both the lines.

12.LINEAR PROGRAMMING

March 2025

Multiple Choice Questions (1 Mark)

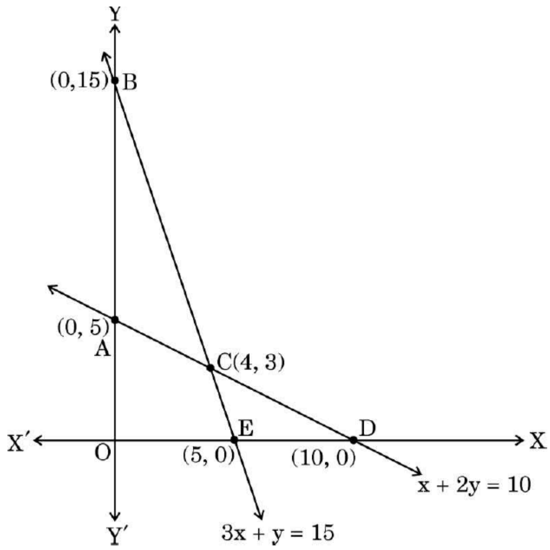
1. The corner points of the feasible region in graphical representation of a L.P.P. are $(2,72)$, $(15,20)$ and $(40,15)$. If $Z = 18x + 9y$ be the objective function, then
(A) Z is maximum at $(2,72)$, minimum at $(15,20)$
(B) Z is maximum at $(15,20)$ minimum at $(40,15)$
(C) Z is maximum at $(40,15)$, minimum at $(15,20)$
(D) Z is maximum at $(40,15)$, minimum at $(2,72)$

2. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ?
(A) It will only have a maximum value.
(B) It will only have a minimum value.
(C) It will have both maximum and minimum values.
(D) It will have neither maximum nor minimum value.

3. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct?
(A) The objective function maximizes the difference of the profit earned from products X and Y.
(B) The objective function measures the total production of products X and Y.
(C) The objective function maximizes the combined profit earned from selling X and Y.
(D) The objective function ensures the company produces more of product X than product Y.

4. The corner points of the feasible region of a Linear Programming Problem are $(0,2)$, $(3,0)$, $(6,0)$, $(6,8)$ and $(0,5)$. If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at $(0,2)$ and $(3,0)$, then the relation between a and b is :
(A) $a = b$
(B) $a = 3b$
(C) $b = 6a$
(D) $3a = 2b$

5. For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints : $x + 2y \leq 10$ $3x + y \leq 15$ $x, y \geq 0$



The correct feasible region is :
(A) ABC
(B) AOEC
(C) CED
(D) Open unbounded region BCD

6. In an LPP, corner points of the feasible region determined by the system of linear constraints are $(1,1)$, $(3,0)$ and $(0,3)$. If $Z = ax + by$, where $a, b > 0$ is to be minimized, the condition on a and b, so that the minimum of Z occurs at $(3,0)$ and $(1,1)$, will be :
(A) $a = 2b$
(B) $a = \frac{b}{2}$
(C) $a = 3b$
(D) $a = b$

7. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 1$, $x, y \geq 0$ is :
(A) 3

(B) 4
(C) 7
(D) 0

8. Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies :
(A) inside the feasible region
(B) at the boundary line of the feasible region
(C) at the corner points of the feasible region
(D) at the coordinate axes

9. The common region for the inequalities $x \geq 0, x + y \leq 1$ and $y \geq 0$, lies in
(A) IV Quadrant
(B) II Quadrant
(C) III Quadrant
(D) I Quadrant

10. The solution set of the inequality $5x + 4y < 7$ is :
(A) Open half-plane containing the origin.
(B) Whole xy-plane except the points lying on the line $5x + 4y = 7$.
(C) Open half-plane not containing the origin.
(D) Closed half-plane not containing the origin.

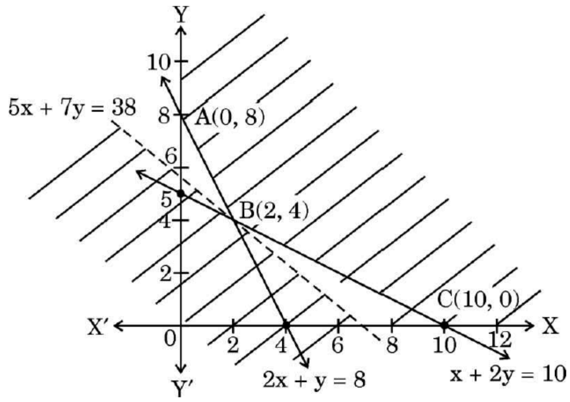
11. Of all the points of the feasible region of an LPP, for maximum or minimum values of objective function, the points lie :
(A) inside the feasible region
(B) at the boundary line of the feasible region
(C) at the corners of the feasible region
(D) at the points of intersection of the feasible region with x-axis

Assertion - Reason Questions (1 Mark)

12. Assertion (A): Every point of the feasible region of a Linear Programming Problem is an optimal solution.
Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true but Reason (R) is false.
(D) Assertion (A) is false but Reason (R) is true.

13. Assertion (A) : In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.
Reason (R) : A feasible region is defined as the region that satisfies all the constraints.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

14. Assertion (A) : The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



Min $Z = 50x + 70y$ subject to constraints $2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$ $Z = 50x + 70y$ has a minimum value $= 380$ at $\text{B}(2, 4)$.
Reason (R) : The region representing $50x + 70y < 380$ does not have any point common with the feasible region.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is

false.
(D) Assertion (A) is false, but Reason (R) is true.

Very Short Answer Questions (2 Marks)

15. In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6, x \leq 1, x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.

Short Answer & Long Answer Questions (3 - 5 Marks)

16. Solve the following linear programming problem graphically : Maximise $Z = x + 2y$ Subject to the constraints : $x - y \geq 0$ $x - 2y \geq -2$ $x \geq 0, y \geq 0$

17. Solve the following linear programming problem graphically: Minimise $Z = x - 5y$ subject to the constraints: $x - y \geq 0$ $-x + 2y \geq 2$ $x \geq 3, y \leq 4, y \geq 0$

18. Solve the following Linear Programming Problem using graphical method : Maximise $Z = 100x + 50y$ subject to the constraints $3x + y \leq 600$ $x + y \leq 300$ $y \leq x + 200$ $x \geq 0, y \geq 0$

19. In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$ subject to constraints $x + 2y \leq 120$ $x + y \geq 60$ $x - 2y \geq 0$ $x, y \geq 0$

20. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints $4x + y \geq 20$ $2x + 3y \geq 30$ $x, y \geq 0$ find the minimum value of Z.

21. Solve the following LPP graphically : Maximize $Z = 2x + 3y$ subject to the constraints $x + 4y \leq 8$ $2x + 3y \leq 12$ $3x + y \leq 9$ $x \geq 0, y \geq 0$.

22. The corner points of the feasible region determined by some system of linear inequations, are $(0, 0)$, $(5, 0)$, $(3, 4)$ and $(0, 5)$. Let $Z = ax + by$, where $a, b > 0$. Find the condition on a and b so that the maximum of Z occurs at both points $(3, 4)$ and $(0, 5)$.

23. The corner points of the feasible region determined by the system of linear constraints for an LPP are $(0, 10)$, $(5, 5)$, $(15, 15)$ and $(0, 20)$. Let $z = ax + by$, where $a, b > 0$ be the objective function. Find the condition on a and b so that maximum of z occurs at both the points $(15, 15)$ and $(0, 20)$.

MARCH 2024

Section A: Multiple Choice Questions (1 Mark)

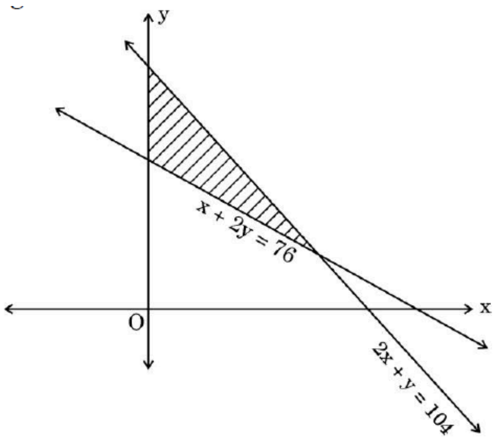
1. A linear programming problem deals with the optimization of a/an :
(A) logarithmic function
(B) linear function
(C) quadratic function
(D) exponential function

2. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :
(A) 0
(B) 1
(C) 2
(D) 3

3. The common region determined by all the constraints of a linear programming problem is called :
(A) an unbounded region
(B) an optimal region
(C) a bounded region
(D) a feasible region

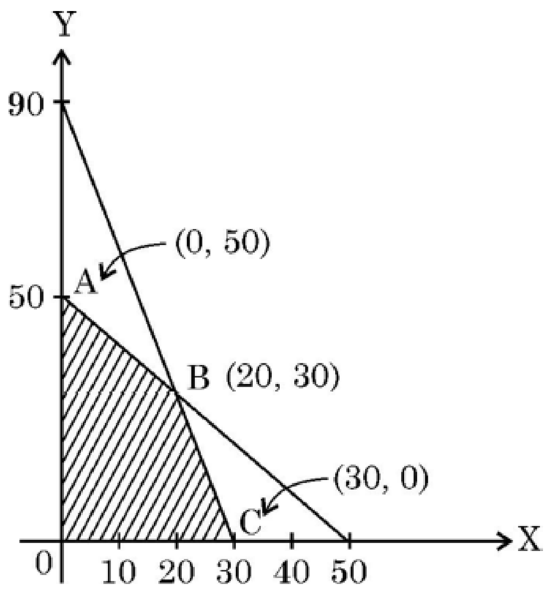
4. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :
(A) feasible solutions
(B) constraints
(C) optimal solutions
(D) infeasible solutions

5. Of the following, which group of constraints represents the feasible region given below ?



(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$
(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$
(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$
(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

6. The maximum value of $Z = 4x + y$ for a L.P.P. whose feasible region is given below is :



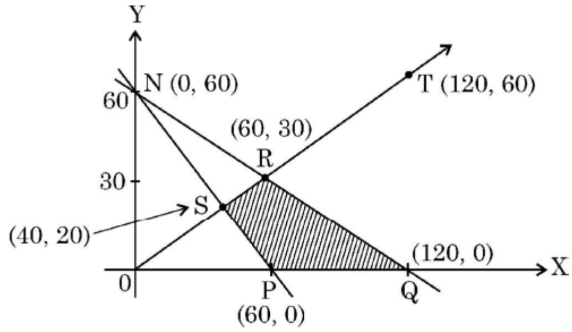
(A) 50
(B) 110
(C) 120
(D) 170

7. The solution set of the inequation $2x + 3y < 6$ is :
(A) open half-plane not containing origin
(B) whole xy-plane except the points lying on the line $2x + 3y = 6$
(C) open half-plane containing origin
(D) half-plane containing the origin and the points lying on the line $2x + 3y = 6$

8. The maximum value of the objective function $z = 3x + 5y$ subject to the constraints $x \geq 0, y \geq 0$ and $4x + 3y \leq 12$ is :
(A) 15
(B) 29
(C) 9
(D) 20

Section A: Assertion-Reason (1 Mark)

9. Assertion (A) : The corner points of the bounded feasible region of a L.P.P. are shown below. The maximum value of $Z = x + 2y$ occurs at infinite points.



Reason (R) : The optimal solution of a LPP having bounded feasible region must occur at corner points.
(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Section C: Short Answer Type Questions (3 Marks)

10. Solve the following linear programming problem graphically : Maximise $z = 500x + 300y$, subject to constraints $x + 2y \leq 12$ $2x + y \leq 12$ $4x + 5y \geq 20$ $x \geq 0, y \geq 0$

11. Solve the following linear programming problem graphically : Maximise $z = 4x + 3y$, subject to the constraints $x + y \leq 800$ $2x + y \leq 1000$ $x \leq 400$ $x, y \geq 0$.

12. Solve the following linear programming problem graphically : Maximise $Z = 2x + 3y$ subject to the constraints : $x + y \leq 6$ $x \geq 2$ $y \leq 3$ $x, y \geq 0$

13. The corner points of the feasible region determined by the system of linear constraints are A$(0, 40)$, B$(20, 40)$, C$(60, 20)$ and D$(60, 0)$. The objective function of the L.P.P. is $z = 4x + 3y$. Find the point of the feasible region at which the value of objective function is maximum and the point at which the value is minimum. Hence, find the maximum and the minimum values.

Section D: Long Answer Type Questions (5 Marks)

14. Solve the following L.P.P. graphically : Maximise $Z = 60x + 40y$ Subject to $x + 2y \leq 12$ $2x + y \leq 12$ $4x + 5y \geq 20$ $x, y \geq 0$

15. Solve the following linear programming problem graphically : Minimise $Z = 6x + 7y$ subject to constraints $x + 2y \geq 240$ $3x + 4y \leq 620$ $2x + y \geq 180$ $x, y \geq 0$.

13. PROBABILITY

March 2025

Multiple Choice Questions (1 Mark)

1. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\ |(F))$ is equal to :
(A) $\frac{1}{6}$
(B) $\frac{1}{2}$
(C) $\frac{2}{3}$
(D) $\frac{7}{9}$

2. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(|(E)/\ |(F))$ is
(A) $\frac{P(E)}{P(F)}$
(B) $1 - P(|(E)/F)$
(C) $1 - P(E/F)$
(D) $\frac{1-P(E\cup F)}{P(|(F))}$

3. If $P(A \cup B) = 0.9$ and $P(A \cap B) = 0.4$, then $P(|(A)) + P(|(B))$ is :
(A) 0.3
(B) 1
(C) 1.3
(D) 0.7

4. If $P(A) = \frac{1}{7}$, $P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(|(A) \mid B)$ is :
(A) $\frac{6}{7}$
(B) $\frac{3}{4}$
(C) $\frac{4}{5}$
(D) $\frac{1}{5}$

5. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :
(A) $\frac{2}{13}$
(B) $\frac{3}{26}$
(C) $\frac{19}{26}$
(D) $\frac{3}{13}$

6. A coin is tossed three times. The probability of getting at least two heads is :
(A) $\frac{1}{2}$
(B) $\frac{3}{8}$
(C) $\frac{1}{8}$
(D) $\frac{1}{4}$

7. A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is :
(A) $\frac{11}{15}$
(B) $\frac{7}{15}$
(C) $\frac{8}{15}$
(D) $\frac{1}{5}$

8. If $P(A) = 0.3, P(B) = 0.4$ and $P(A \cup B) = 0.6$, then $P(A \mid B) + P(B \mid A)$ equals :
(A) $\frac{5}{12}$
(B) $\frac{1}{4}$
(C) $\frac{1}{3}$
(D) $\frac{7}{12}$

Assertion - Reason Questions (1 Mark)

9. **Assertion (A) :** If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.
Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Very Short Answer Questions (2 Marks)

10. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?

Short Answer & Long Answer Questions (3 - 5 Marks)

11. (a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B: $A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space. Check if events A and B are independent or mutually exclusive.

12. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

13. (a) The probability that a student buys a colouring book is 0.7 and that she buys a box of colours is 0.2. The probability that she buys a colouring book, given that she buys a box of colours, is 0.3. Find the probability that the student : (i) Buys both the colouring book and the box of colours. (ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find : (i) The probability distribution of the number of oranges he draws. (ii) The expectation of the random variable (number of oranges).

14. (a) Find the probability distribution of the number of doublets in three throws of a pair of dice.

OR

(b) If E and F are two independent events with $P(E) = p$, $P(F) = 2p$ and $P(\text{exactly one of E, F}) = \frac{5}{9}$, then find the value of p.

15. (a) Two balls are drawn at random without replacement from a box containing 3 black and 7 red balls. Find the probability that : (i) both balls are red. (ii) first ball is black and the second is red.

OR

(b) Two cards are drawn successively with replacement of a well-shuffled pack of 52 playing cards. Find the probability distribution of the number of face cards.

Case Study Based Questions (4 Marks)

16. A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively. Based on the above information, answer the following : (i) What is the probability that a customer after availing the loan will default on the loan repayment ? **2** (ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest ? **2**

17. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated). Based on the above, answer the following: (i) (a) What is the probability that a randomly selected car is an electric car? **2**

OR

(i) (b) What is the probability that a randomly selected car is a petrol car? **2** (ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet? **1** (iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi? **1**

18. Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43.9
Devesh	45.2

The students were asked to identify who has thrown the javelin the farthest. Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions : (i) What is the probability of a student not having misconception but still answers Bijoy in the test ? **1** (ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? **1** (iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? **2**

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? **2**

19. A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.

Based upon the above information, answer the following questions : (i) Calculate the probability of a randomly chosen seed to germinate. (ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates?

20. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record. Let A_1 : People with good health, A_2 : People with average health, and A_3 : People with poor health. During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively. Based upon the above information, answer the following questions : (i) A person was tested randomly. What is the probability that he/she has contracted the disease ? **2** (ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? **2**

21. Two persons are competing for a position on the Managing Committee of an organisation. The probabilities that the first and the second person will be appointed are 0.5

and 0.6 respectively. Also, if the first person gets appointed, then the probability of introducing waste treatment plant is 0.7 and the corresponding probability is 0.4, if the second person gets appointed. Based on the above information, answer the following questions : (i) What is the probability that the waste treatment plant is introduced ? **2** (ii) After the selection, if the waste treatment plant is introduced, what is the probability that the first person had introduced it ? **2**

22. There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002, of category B it is 0.02 and of category C, this probability is 0.20. Based on the above information, answer the following : (i) Find the probability that a student selected at random is unable to get good marks in the final examination. **2** (ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is **NOT** of category A. **2**

23. A coach is training 3 players. He observes that player A can hit the target 4 times in 5 shots, player B can hit the target 3 times in 4 shots and player C can hit the target 2 times in 3 shots. Based on the above information, answer the following questions : If they all try independently, find the probability that : (i) exactly two of them hit the target. **2** (ii) at least one of them hits the target. **2**

March 2024

Section A: Multiple Choice Questions (1 Mark)

1. If $P(A \mid B) = P(A' \mid B)$, then which of the following statements is true ?
(A) $P(A) = P(A')$
(B) $P(A) = 2P(B)$
(C) $P(A \cap B) = \frac{1}{2}P(B)$
(D) $P(A \cap B) = 2P(B)$

2. Let E be an event of a sample space S of an experiment, then $P(S \mid E) =$
(A) $P(S \cap E)$
(B) $P(E)$
(C) 1
(D) 0

3. Let E and F be two events such that $P(E) = 0.1$, $P(F) = 0.3$, $P(E \cup F) = 0.4$, then $P(F \mid E)$ is :
(A) 0.6
(B) 0.4
(C) 0.5
(D) 0

4. If A and B are events such that $P(A/B) = P(B/A) \neq 0$, then :
(A) $A \subset B$, but $A \neq B$
(B) $A = B$
(C) $A \cap B = \varphi$
(D) $P(A) = P(B)$

5. The probabilities of A, B and C solving a problem are $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{6}$ respectively. The probability that the problem is solved, is :
(A) $\frac{4}{9}$
(B) $\frac{5}{9}$
(C) $\frac{1}{90}$
(D) $\frac{1}{3}$

Section A: Assertion-Reason (1 Mark)

6. (Assertion-Reason)
Assertion (A) : If R and S are two events such that $P(R \mid S) = 1$ and $P(S) > 0$, then $S \subset R$.
Reason (R) : If two events A and B are such that $P(A \cap B) = P(B)$, then $A \subset B$.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

Section C: Short Answer Type Questions (3 Marks)

7. E and F are two independent events such that $P(|(E)) = 0.6$ and $P(E \cup F) = 0.6$. Find $P(F)$ and $P(|(E) \cup |(F))$.

8. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

9. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0.3, 0.8 and 0.5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

10. (a) A card from a well shuffled deck of 52 playing cards is lost. From the remaining cards of the pack, a card is drawn at random and is found to be a King. Find the probability of the lost card being a King.

OR

(b) A biased die is twice as likely to show an even number as an odd number. If such a die is thrown twice, find the probability distribution of the number of sixes. Also, find the mean of the distribution.

11. (a) A card is randomly drawn from a well-shuffled pack of 52 playing cards. Events A and B are defined as under : A : Getting a card of diamond B : Getting a queen Determine whether the events A and B are independent or not.

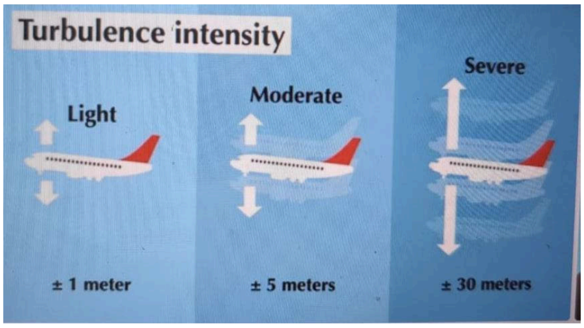
OR

(b) Find the probability distribution of the number of doublets in three throws of a pair of dice.

12. It is known that 20% of the students in a school have above 90% attendance and 80% of the students are irregular. Past year results show that 80% of students who have above 90% attendance and 20% of irregular students get 'A' grade in their annual examination. At the end of a year, a student is chosen at random from the school and is found to have an 'A' grade. What is the probability that the student is irregular ?

Section E: Case Study Based Questions (4 Marks)

13. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights. Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions : (i) Find the probability that an airplane reached its destination late. (ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

14. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals. Previous records state that the probability of an airplane crash is 0.00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travelers survive. Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey. On the basis of the above information, answer the following questions : (i) Find the probability that the airplane will not crash. (ii) Find $P(A \mid E_1) + P(A \mid E_2)$. (iii) (a) Find $P(A)$.

OR

(iii) (b) Find $P(E_2 \mid A)$.

15. Rohit, Jaspreet and Alia appeared for an interview for three vacancies in the same post. The probability of Rohit's selection is $\frac{1}{5}$, Jaspreet's selection is $\frac{1}{3}$ and Alia's selection is $\frac{1}{4}$. The event of selection is independent of each other. Based on the above information, answer the following questions : (i) What is the probability that at least one of them is selected ? (ii) Find $P(G \mid |(H))$ where G is the event of Jaspreet's selection and $|(H)$ denotes the event that Rohit is not selected. (iii) Find the probability that exactly one of them is selected.

OR

(iii) Find the probability that exactly two of them are selected.

16. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month. Based on the above information, answer the following questions : (i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. Find $P(E_1), P(E_2)$. (ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A \mid E_1)$ and $P(A \mid E_2)$. (iii) Find the probability of customer paying second month's bill in time.

OR

(iii) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

17. A coach is training 3 players. He observes that player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and player C can hit 2 times in 3 shots. Based on the above, answer the following questions : (i) Find the probability that all three players miss the target. (ii) Find the probability that all of them hit the target. (iii) (a) Find the probability that only one of them hits the target.

OR

(iii) (b) Find the probability that exactly two of them hit the target.

1. RELATIONS & FUNCTIONS

March, 2025

Multiple Choice Questions (1 Mark)

1. **(C)** f is one-one and onto on $\mathbb{R}$.
2. **(B)** surjective only (because $f(1) = f(3) = 0$, not injective; range is W, so surjective).
3. **(C)** $f(x) = x + 5$
4. **(C)** 4 (Formula: $2^{n^2-n} = 2^{4-2} = 2^2 = 4$)

Assertion - Reason Questions (1 Mark)

5. **(D)** Assertion (A) is false, but Reason (R) is true. (Note: $f(x) = 3x - 5$ is not surjective on $\mathbb{Z}$ because $y = 0$ gives $x = \frac{5}{3} \notin \mathbb{Z}$).
6. **(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation.
7. **(C)** Assertion (A) is true, but Reason (R) is false (Reason defines 'onto', not 'one-one').

Very Short Answer Questions (2 Marks)

8. **Solution:**
   - **One-one:** Let $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$. Cross-multiplying: $(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$
     $x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6$
     $-x_1 = -x_2 \Rightarrow x_1 = x_2$. So, it is **one-one**.
   - **Onto:** Let $y = \frac{x-2}{x-3} \Rightarrow yx - 3y = x - 2 \Rightarrow x(y-1) = 3y - 2 \Rightarrow x = \frac{3y-2}{y-1}$. Since $y \in B(\mathbb{R} - \{1\})$, x is always defined and $x \in A$. So, it is **onto**.
   - **Conclusion:** The function is **bijective**.

Short Answer & Long Answer Questions (3 - 5 Marks)

9. **Solution:**
   - **Reflexive:** $mRm \Rightarrow m$ is a multiple of m, which is true $\forall m \in \mathbb{N}$. (Reflexive)
   - **Symmetric:** If $4R2$ (4 is a multiple of 2), then $2R4$ (2 is a multiple of 4) is false. (Not Symmetric)
   - **Transitive:** If mRn $(m = kn)$ and nRp $(n = jp)$, then $m = k(jp) = (kj)p$, so m is a multiple of p. (Transitive)

10. (a) **Solution:**
    - **One-one:** $f(x_1) = f(x_2) \Rightarrow \log_a x_1 = \log_a x_2 \Rightarrow x_1 = x_2$.
    - **Onto:** For any $y \in \mathbb{R}$, let $y = \log_a x \Rightarrow x = a^y$. Since $a > 0$, a^y is always a positive real number $(x \in \mathbb{R}^+)$.
    - Hence, f is a **bijection**.

    OR

    (b) **Solution:**
    (i) $R = \{(1,5), (2,4)\}$. (Note: $3 + 3 = 6$ but $3 \notin B$) (ii) **No**, R is not a function because the element $3 \in A$ does not have an image in B. (iii) Domain $= \{1, 2\}$, Range $= \{4, 5\}$.

11. (a) **Solution:**
    - **One-one:** $f(x_1) = f(x_2) \Rightarrow 4x_1^3 - 5 = 4x_2^3 - 5 \Rightarrow x_1^3 = x_2^3 \Rightarrow x_1 = x_2$.
    - **Onto:** Let $y = 4x^3 - 5 \Rightarrow x = \sqrt[3]{\frac{y+5}{4}}$. For every $y \in \mathbb{R}$, $x \in \mathbb{R}$.

    OR

    (b) **Solution:**
    - **Reflexive:** $x \cdot x = x^2$ (a square), so $(x, x) \in R$.
    - **Symmetric:** $xy = k^2 \Rightarrow yx = k^2$, so $(y, x) \in R$.
    - **Transitive:** $xy = k^2$ and $yz = m^2$. Then $(xy)(yz) = (km)^2 \Rightarrow y^2(xz) = (km)^2 \Rightarrow xz = \left(\frac{km}{y}\right)^2$. Since xz is a square of a rational number and x, z are integers, xz must be a square of a natural number.
    - **Conclusion:** It is an **equivalence relation**.

12. **Solution:**
    - **Reflexive:** $(a,b)R(a,b) \Rightarrow ab = ba$ (True).
    - **Symmetric:** $(a,b)R(c,d) \Rightarrow ad = bc \Rightarrow cb = da \Rightarrow (c,d)R(a,b)$.
    - **Transitive:** $(a,b)R(c,d)$ and $(c,d)R(e,f) \Rightarrow ad = bc$ and $cf = de$. Multiply: $(ad)(cf) = (bc)(de) \Rightarrow af = be \Rightarrow (a,b)R(e,f)$.
    - Hence, it is an **equivalence relation**.

Case Study Based Questions (4 Marks)

13. Solution:

(i) R_4
(ii) R_5 (Reflexive/Symmetric, but $(1,2),(2,3) \in R_5$ while $(1,3) \notin R_5$)
(iii) (a) R_1
(iii)
(b) $\{(1,1),(2,2),(3,3),(2,1),(2,3),(3,1)\}$

14. Solution:

(i) Total relations $= 2^{4\times 3} = 2^{12} = 4096$. (ii) **Not bijective**. It is not one-one because $f(S_2) = J_2$ and $f(S_3) = J_2$. (iii) (a) **0**. (Since $|S| > |J|$, no one-one function is possible). (iii) (b) Add: $\{(S_1,S_1),(S_2,S_2),(S_3,S_3),(S_4,S_4)\}$.

15. Solution:

(i) Yes. (ii) One-one: No two students have the same roll number. Onto: Every roll number from 1 to 30 is assigned to exactly one student. (iii) (a) $R = \{(1,3),(2,6),(3,9),(4,12),(5,15),(6,18),(7,21),(8,24),(9,27),(10,30)\}$. Not reflexive ($(1,1) \notin R$), not symmetric, not transitive. (iii) (b) $R = \{(1,1),(2,8),(3,27)\}$. Yes, it is a function as every input has a unique output.

16. Solution:

(i) Range $= \{1,4,9,16,25,...\}$ (Set of perfect squares). (ii) **Injective**. $x_1^2 = x_2^2 \Rightarrow x_1 = x_2$ for $x \in \mathbb{N}$. (iii) (a) It is one-one ($x_1^2 = x_2^2 \Rightarrow x_1 = x_2$) and onto (codomain is exactly the set of squares). Hence, bijective. (iii) (b) Not injective: $f(1) = f(-1) = 1$. Not surjective: negative numbers in codomain $\mathbb{R}$ have no pre-image.

17. Solution:

(i) Not one-one ($f(1) = f(-1)$). (ii) Not onto (Negative numbers have no pre-image). (iii) (a) One-one: Yes. Onto: No (e.g., 2 is not a square of any natural number). (iii) (b) One-one: Yes. Onto: Yes.

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Section A: Multiple Choice Questions (1 Mark)

1. **(A)** one-one but not onto (Range is $[3,\infty)$, not $\mathbb{R}$).

2. **(C)** bijective.

3. **(A)** one-one but not onto.

4. **(C)** Assertion (A) is true, but Reason (R) is false (2 is prime, but $2n$ is not composite for $n = 1$).

5. **(D)** neither injective nor surjective.

Section C: Short Answer Type Questions (3 Marks)

6. (a) **Solution:**
   - Reflexive: $|x^2 - x^2| = 0 < 8$. **Yes**.
   - Symmetric: $|x^2 - y^2| = |y^2 - x^2|$. **Yes**.
   - Transitive: $(1,2) \in R$ $(|1-4| = 3)$ and $(2,3) \in R$ $(|4-9| = 5)$, but $(1,3) \notin R$ $(|1-9| = 8 \not< 8)$. **No**.

OR

(b) **Solution:**
$f(1) = a + b = 1$; $f(2) = 2a + b = 3$. Solving gives $a = 2, b = -1$. So $\boldsymbol{f(x) = 2x - 1}$. It is a linear function, hence **one-one and onto** on $\mathbb{R}$.

Section D: Long Answer Type Questions (5 Marks)

7. (a) **Solution:**
   - Not one-one: $f(2) = \frac{4}{5}$ and $f\left(\frac{1}{2}\right) = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$.
   - Not onto: $y = \frac{2x}{1+x^2} \Rightarrow yx^2 - 2x + y = 0$. For real x, $D \geq 0 \Rightarrow 4 - 4y^2 \geq 0 \Rightarrow y^2 \leq 1$. Range is $[-1,1] \neq \mathbb{R}$.
   - Set $A = [-1,1]$.

OR

(b) **Solution:**
   - Reflexive: $a - a = b - b \Rightarrow (a,b)R(a,b)$.
   - Symmetric: $a - c = b - d \Rightarrow c - a = d - b \Rightarrow (c,d)R(a,b)$.

- Transitive: $a-c=b-d$ and $c-e=d-f$. Adding them: $a-e=b-f \Rightarrow (a,b)R(e,f)$.

8. Solution:
Reflexivity, Symmetry, and Transitivity follow standard parity rules (Sum of two integers is even if both are even or both are odd). Equivalence class **[2] = {-4, −2, 0, 2, 4}** (all even numbers in set A).

9. (a) **Solution:**
- One-one: $f(x_1)=f(x_2) \Rightarrow \frac{x_1-3}{x_1-5}=\frac{x_2-3}{x_2-5} \Rightarrow \ldots \Rightarrow x_1=x_2$.
- Onto: $y=\frac{x-3}{x-5} \Rightarrow x=\frac{5y-3}{y-1}$. Since $y\neq 1$, x is defined for all B.

OR

(b) **Solution:**
- Reflexive: $a-a+\sqrt{2}=\sqrt{2}$ (Irrational). **Yes**.
- Symmetric: Let $a=\sqrt{2}, b=0$. $a-b+\sqrt{2}=2\sqrt{2}$ (Irr). But $b-a+\sqrt{2}=0-\sqrt{2}+\sqrt{2}=0$ (Rat). **No**.
- Transitive: **No**.

10. (a) **Solution:**
Equivalence relation proof: $a-a=0$ (div by 4); $a-b=4k \Rightarrow b-a=-4k$; $(a-b)+(b-c)=4k+4m$. Elements related to 2: **{2, 6, 10}**.

11. Solution:
- Reflexive: $\frac{1}{2}\not\leq\left(\frac{1}{2}\right)^3$. **No**.
- Symmetric: $1\leq 2^3$, but $2\not\leq 1^3$. **No**.
- Transitive: $10\leq 3^3$ and $3\leq 2^3$, but $10\not\leq 2^3$. **No**.

Section E: Case Study Based Questions (4 Marks)

12. (a) **Solution:**
(i) Symmetric: Yes, if $l_1 \parallel l_2$, then $l_2 \parallel l_1$. (ii) Transitive: Yes, if $l_1 \parallel l_2$ and $l_2 \parallel l_3$, then $l_1 \parallel l_3$. (iii) Set of lines $y=3x+c$, where $c\in\mathbb{R}$.

OR

(b) **Solution:**
Symmetric: **Yes** ($l_1\perp l_2 \Rightarrow l_2 \perp l_1$). Transitive: **No** ($l_1\perp l_2$ and $l_2\perp l_3 \Rightarrow l_1 \parallel l_3$).

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1. (b) 4

2. Solution: $B=f(A)=\{2(1),2(2),2(3),2(4)\}=\{2,4,6,8\}$.

3. Solution:
- Injectivity: $x_1^3=x_2^3 \Rightarrow x_1=x_2$. **Yes**.
- Surjectivity: There is no $x\in\mathbb{N}$ such that $x^3=2$. **No**.

4. Solution:
- One-one: $f(1.1)=1, f(1.2)=1$. Not one-one.
- Onto: Range is $\mathbb{Z}$ (integers), but codomain is $\mathbb{R}$. Not onto.

5. (a) **Solution:**
$x=12-3y$. For $y=1,x=9; y=2,x=6; y=3,x=3$. Domain = **{3, 6, 9}**, Range = **{1, 2, 3}**.

OR

(b) **Solution:**
One-one: $f(1)=1, f(-1)=1$. **No**. Onto: $x^4=-1$ has no real solution. **No**.

6. Solution:
- Reflexive: $1\cdot 1=1$ (Rational). **No**.
- Symmetric: xy is irrational $\Rightarrow yx$ is irrational. **Yes**.
- Transitive: $\sqrt{2}\cdot 1$ is Irr, $1\cdot\sqrt{2}$ is Irr, but $\sqrt{2}\cdot\sqrt{2}=2$ (Rat). **No**.

7. Solution:
- Onto: For $y\in[0,4]$, $x=\sqrt{16-y^2}$ which is in $[-4,4]$.
- One-one: $f(1)=\sqrt{15}, f(-1)=\sqrt{15}$. **No**.
- $f(a)=\sqrt{7} \Rightarrow 16-a^2=7 \Rightarrow a^2=9 \Rightarrow a=\pm 3$.

8. Solution:
$f(x_1)=f(x_2) \Rightarrow 5x_1-3=5x_2-3 \Rightarrow x_1=x_2$ (One-one). $y=\frac{5x-3}{4} \Rightarrow x=\frac{4y+3}{5}$. Since $x\in\mathbb{R}$ for all $y\in\mathbb{R}$ (Onto).

9. (a) **Solution:**
Rewrite: $ad(b+c)=bc(a+d) \Rightarrow \frac{b+c}{bc}=\frac{a+d}{ad} \Rightarrow \frac{1}{c}+\frac{1}{b}=\frac{1}{d}+\frac{1}{a}$. This symmetry makes

Reflexive, Symmetric, and Transitive properties obvious.

OR

(b) **Solution:**
$f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4} \Rightarrow 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2 \Rightarrow x_1 = x_2$. (One-one).
Onto check: $y = \frac{4x}{3x+4} \Rightarrow x = \frac{4y}{4-3y}$. If $y = \frac{4}{3}$, x is not defined. Since codomain is $\mathbb{R}$, it is **not onto**.

10. (a) **Solution:**
$R = \{(5,5), (5,7), (5,9), (6,6), (6,8), (7,5), (7,7), (7,9), (8,6), (8,8), (9,5), (9,7), (9,9)\}$.
Elements related to 7: **{5, 7, 9}**.

OR

(b) **Solution:**
$y = \frac{x-2}{x-3} \Rightarrow x = \frac{3y-2}{y-1}$. Since $y \in B(\mathbb{R} - \{1\})$, x is always defined. **Onto**. One-one: Yes (standard linear fractional proof).

11. (a) Repeat of 2024 Question 11.

OR

(b) **Solution:** Equivalence class [1] = **{1, 3, 5, 7}** (all odd numbers).

12. (a) Repeat of 2024 Question 11.

OR

(b) **Solution:** $a + d = b + c \Rightarrow a - b = c - d$. Proof follows same steps as Q7(b) March 2024.

13. Case Study Solution:
(I) $2^{3\times2} = 2^6 = 64$. (II) $2^3 = 8$. (III) **Yes**, it is an equivalence relation (Every student is same sex as themselves; if x, y same sex, then y, x same sex; etc).

OR

(III) **No**, $f(b_1) = g_1$ and $f(b_3) = g_1$, so it is not one-one. Thus, not bijective.

Answers

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1. (B) (Reflection across $y = x$).

2. (B) $[-\frac{\pi}{2}, 0]$

3. (C) $\frac{\pi}{3}$

4. (D) $\frac{2\pi}{3}$

5. (B) $y = \operatorname{cosec}^{-1} x$

6. (C) $[-1, 1]$

7. (D) $[-\frac{1}{2}, \frac{1}{2}]$

8. (A) 3

9. (A) Both true, R explains A.

10. Ans: $\frac{\pi}{3}$

11. (a) $\tan^{-1} x$ **OR** (b) $[1, 2]$

12. Ans: $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$

13. (a) π **OR** (b) Proof.

14. Ans: $-\frac{\pi}{3}$

15. Ans: 15

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1. (C) A true, R false.

2. (A) Both true, R explains A.

3. (A) Both true, R explains A.

4. (a) $-\frac{\pi}{12}$ **OR** (b) D: $\left[-\sqrt{5}, -\sqrt{3}\right] \cup \left[\sqrt{3}, \sqrt{5}\right]$, R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

5. Ans: $\frac{85}{36}$

6. (a) $\frac{\pi}{4} + \frac{x}{2}$ **OR** (b) $\frac{2\pi}{3}$

7. Ans: $k = \frac{1}{2}$

8. (a) $\frac{3\pi}{4}$ **OR** (b) As Q4(b)

9. Ans: $\frac{\pi}{3}$

10. (i) $[\frac{\pi}{2}, \frac{3\pi}{2}]$ (ii) $-\frac{2\pi}{3}$ (iii) D: $[0, 2]$, R: $[-\pi, \pi]$

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1. (a) 1

2. (c) $[-1, 1]$

3. (c) $[-\frac{1}{2}, \frac{1}{2}]$

4. (c) A true, R false

5. (c) A true, R false

6. (d) A false, R true

7. (d) A false, R true

8. (c) A true, R false

9. Ans: $\frac{5\pi}{4}$

10. (a) $\frac{3\pi}{2}$ **OR** (b) $[\frac{\pi}{2}, \pi]$

11. (a) $\frac{19\pi}{12}$ **OR** (b) $[-\frac{\pi}{4}, \frac{\pi}{4}]$

12. (a) As Q4(b) **OR** (b) $\frac{\pi}{3}$

13. D: $\mathbb{R}$, R: $(-\frac{\pi}{2}, \frac{\pi}{2})$

14. Ans: $-\frac{\pi}{3}$

15. Ans: $\frac{\pi}{4} + \frac{x}{2}$

3 & 4. MATRICES & DETERMINANTS

March, 2025

Multiple Choice Questions (1 Mark)

1. **(D)** $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (For a diagonal matrix, $(\text{diag}[a, b, c])^{-1} = \text{diag}\left[\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right]$).

2. **(B)** 64 (Formula: $|k \text{ adj } A| = k^n \; |A|^{n-1} = 4^2(4)^1 = 16 \times 4 = 64$).

3. **(A)** Only AB (3×3 times 3×1 is defined).

4. **(B)** 1 (Scalar matrix $\Rightarrow x = 0, y = 7$. $7^0 = 1$).

5. **(A)** $(A+B)^{-1} = B^{-1} + A^{-1}$ (This property is false for addition).

6. **(D)** 8 ($6x = 12 \Rightarrow x = 2$; $4y = 8x \Rightarrow 4y = 16 \Rightarrow y = 4$. $2(2) + 4 = 8$).

7. **(C)** Null Matrix.

8. **(B)** Bina (Expansion: $4AB + 3AB + 3BA - 4BA = 7AB - BA$).

9. **(B)** $AB = BA$ (Expansion of RHS yields $A^2 + AB - BA - B^2$).

10. **(A)** $A + B$ (Since $A^2 = A(BA) = (AB)A = A^2$, $A^2 = A$ and $B^2 = B$).

11. **(D)** m^2 ($|MN| = |mI| \Rightarrow m|N| = m^3 \Rightarrow |N| = m^2$).

12. **(C)** skew symmetric matrix (Diagonals are 0 and $a_{ij} = -a_{ji}$).

13. **(B)** $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$ ($A = 5I \Rightarrow A^3 = 125I$).

14. **(C)** AB and BA, both are defined ($2 \times 3 \cdot 3 \times 2$ and $3 \times 2 \cdot 2 \times 3$).

15. **(C)** ± 7 ($2x^2 - 60 = 18 - (-20) \Rightarrow 2x^2 = 98 \Rightarrow x^2 = 49$).

16. **(B)** -10 (Determinant of diagonal matrix is product of diagonal elements).

17. **(C)** -2 (Solve $(4 + x)3 - (-2)(x - 1) = 0 \Rightarrow 12 + 3x + 2x - 2 = 0$).

18. **(B)** 512 (2^9).

19. **(D)** symmetric matrix (Also a diagonal matrix).

20. **(B)** 120 ($|2AB| = 2^3 \cdot 3 \cdot 5 = 120$).

21. **(C)** 25 ($|A|^{3-1} = 5^2$).

22. **(B)** -2 or 10 ($x = 4, y = \pm 6$).

23. **(C)** $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ($A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$).

24. **(B)** zero ($\cos(75 + 15) = \cos 90$).

25. **(A)** X ($X^2 = I \Rightarrow X \cdot X = I \Rightarrow X = X^{-1}$).

26. **(C)** 5 (Minor $M_{32} = 1 - 6 = -5$; Cofactor $C_{32} = -(-5) = 5$).

27. **(A)** identity matrix.

28. **(B)** 12 ($x = 6, z = 6, m = 0, y = 0$).

29. **(D)** 1×2.

30. **(B)** $A^2 - AB - BA + B^2$.

31. **(C)** -6.

32. **(C)** $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$ (Using $A = (A^{-1})^{-1}$).

33. **(B)** ± 4 ($|A| = 7 \Rightarrow x^2 - 9 = 7$).

34. **(B)** -48 ($8 \cdot -3 \cdot 2$).

35. **(A)** 11 ($p = 0, r = -7, q = 4 \Rightarrow 0 + 4 - (-7) = 11$).

36. **(B)** 3.

37. **(D)** 49 ($|A| = 7$, $|\text{adj } A| = 7^2$).

Assertion - Reason Questions (1 Mark)

38. **(D)** Assertion is false (diagonal elements are not equal), Reason is true.

39. (B) Both are true but Reason is not the explanation (The explanation is that for odd order skew-symmetric matrices, $|A| = |-A'| = (-1)^3\ |A| = -|A| \Rightarrow 2|A| = 0$).

Very Short Answer Questions (2 Marks)

40. $|-6AB| = (-6)^3\ |A||B| = -216 \times 3 \times (-4) = \mathbf{2592}$.

41. $A^2 = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$. $A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \boldsymbol{O}$.

Short Answer & Long Answer Questions (3 - 5 Marks)

42. Let x, y, z be Sports, Music, Drama. $x - y - z = 0$; $-x + 2y = 40$; $x + y + z = 180$. Solve $X = A^{-1}B$. Results: **Sports = 90, Music = 65, Drama = 25**.

43. (a) $AB = 8I \Rightarrow A^{-1} = \frac{1}{8}B$. Equations represent $B^T X = D$. Solve to find $x = 3, y = -2, z = -1$.

44. $(3A)^{-1} = \frac{1}{3}A^{-1}$. Find adj A and $|A| = 4$. Result: $\frac{1}{12}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$.

45. $x + y + z = 45, z - x = 8, x + z - 2y = 0$. Solve via matrix: **Chairs = 11, Tables = 15, Beds = 19**.

46. (a) $\left(1, -\frac{1}{5}\right)$ wait, solving $2x + 5y = 1$ and $3x + 2y = 7 \Rightarrow x = 3, y = -1$. Point: **(3, −1)**. (b) Sales: ₹ 29,800 + ₹ 22,875 = ₹ 52,675. **Profit = ₹ 17,675**.

47. $|A| = -20 - 2(26) + 1(19) = -53$ (Expand). Find adj A. Use $X = \left(A^T\right)^{-1}B$ as equations are transpose of A.

48. $|A| = -1$. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$. $x = 1, y = 2, z = 3$.

Case Study Based Questions (4 Marks)

49. (i) $A = \begin{bmatrix} 15 & 10 & 25 \end{bmatrix}$. (ii) $B = \begin{bmatrix} 100 & 150 \\ 150 & 200 \\ 110 & 150 \end{bmatrix}$. (iii) (a) ₹ 5,750 (b) ₹ 8,000.

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Section A (1 Mark)

1. (D) 9.

2. (D) 18 $(24\frac{x+y}{xy} = 24\frac{6}{8})$.

3. (B) 2.

4. (B) $AB = -BA$.

5. (C) 27.

6. (D) 4.

7. (C) $a_{13} > a_{31}$ is False $(1 - 9 = -8$ and $3 - 3 = 0)$.

8. (B) 2.

9. (D) Definition of Identity matrix.

10. (D) 0 (If adj $A = A$, then $d = a, -b = b, -c = c \Rightarrow b, c = 0, d = a$. Also $|A| = a$...).

11. (B) symmetric matrix.

12. (C) ± 2.

13. (A) Diagonals are reciprocals.

14. (D) 25 $(a = 5, d = 5, b = 0, c = 0)$.

15. (B) $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$.

16. (B) $\begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$ (Note $A^2 = O$).

17. (D) 1000 $(|A|^3 = (-10)^3)$.

18. (C) 2 $(4 - 2x = 0)$.

19. (C) -10 $(a = 0, c = 1, b = -5 \Rightarrow 0 - (-5 + 1)$... wait $b = -c, -1 = -1$ no $b = -(-1) = 1, c = -1)$. Result: -10.

20. (D) $2\sqrt{2}$ $(|A|^2 = 8 \Rightarrow |A| = \sqrt{8})$.

21. (D) 4.

22. (A) -1 $(5x - 2 = -6 + x)$.

23. (B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

24. (C) 0.

25. **(C)** $a = -5, b = 7$.

26. **(B)** 2.

27. **(D)** $\begin{bmatrix} 2 & 0 \\ 3 & 7 \end{bmatrix}$.

28. **(B)** 1.

29. **(D)** $\frac{1}{16}B$ $((4A)^{-1} = \frac{1}{4}A^{-1} = \frac{1}{4}(\frac{1}{4}B))$.

30. **(C)** 15 $(m = 3, n = 5)$.

Section A: Assertion-Reason (1 Mark)

31. **(A)** $|A| = 2(1 + \cos^2\theta)$. Since $0 \leq \cos^2\theta \leq 1$, $|A| \in [2, 4]$.

32. **(D)** Assertion is false ($B'AB$ is symmetric if A is symmetric), Reason is true.

33. **(C)** Assertion is true, Reason is false.

Section C (3 Marks)

34. (a) $A = \begin{bmatrix} 17 & 10 \\ 0 & -16 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ -1 & -2 \end{bmatrix}^{-1} = \begin{bmatrix} 3 & -5 \\ -2 & 8 \end{bmatrix}$. $A^{-1} = \frac{1}{14}\begin{bmatrix} 8 & 5 \\ 2 & 3 \end{bmatrix}$. (b) Given A^2, find $A^2 \cdot A^2 = A^4$. If $A^4 = I$, then $A^3 = A^{-1}$.

Section D (5 Marks)

35. (a) $|A| = -1, A^{-1} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$. Solution: $x = 0, y = -5, z = -3$.

36. (a) Let $u = \frac{1}{x}, v = \frac{1}{y}, w = \frac{1}{z}$. Solve linear system: $x = 2, y = 3, z = 5$.

37. $A^{-1} = \frac{1}{10}\begin{bmatrix} 7 & -5 & -4 \\ -11 & 1 & 4 \\ 1 & -3 & 4 \end{bmatrix}$. Note equations are $A^TX = B$. Result: $x = 3, y = 2, z = -1$.

38. (a) $A^{-1} = \frac{1}{9}\begin{bmatrix} 6 & -6 & -6 \\ -3 & 3 & -3 \\ 4 & 0 & -4 \end{bmatrix}$. $x = 1, y = 1, z = 1$ wait... result $x = -1, y = 2, z = 0$. (b) Product $= I$. So $A^{-1} = B$. $x = 3, y = -2, z = 1$.

39. $x = 2, y = 1, z = -1$.

Section E (4 Marks)

40. (i) $x + y = 50, 3x + 4y = 180$. (ii) $|A| = 1 \neq 0$ (Consistent). (iii) (a) Girl = 20, Meritorious = 30. (b) ₹ 1,70,000.

41. (i) $(x - 25)(y + 25) = xy + 625 \Rightarrow x - y = 50$. $(x - 20)(y + 10) = xy - 200 \Rightarrow x - 2y = 0$. (ii) **Length = 100m, Breadth = 50m**.

5. CONTINUITY & DIFFERENTIABILITY

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Multiple Choice Questions (1 Mark)

1. **(C)** $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$. (Modulus functions are continuous everywhere but not differentiable at their roots).

2. **(A)** (Assuming the diagram where set B is inside A, as every differentiable function is continuous).

3. **(A)** $x\frac{dy}{dx}$

4. **(D)** -1 (LHL at $x = 1$ is $3(1) - 2 = 1$; RHL is $2(1)^2 + a(1)$. Equating: $1 = 2 + a \Rightarrow a = -1$).

5. **(B)** $x = 1$ (LHL = 1, RHL = 5).

6. **(C)** ± 1 ($\lim_{x\to 0} \frac{\sin^2 ax}{x^2} = a^2$. For continuity, $a^2 = 1$).

7. **(B)** f is neither continuous nor differentiable at $x = 2$. (GIF is discontinuous at all integers).

8. **(A)** $a = 3, b = -8$ (Equations: $3a + b = 1$ and $5a + b = 7$).

9. **(B)** $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$ (Since $f'(x) = -16x^7$, which is an odd function).

10. **(B)** $-\frac{2}{9}$ ($\frac{dy}{dx} = \frac{2}{3t}$, $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{2}{3t}\right) \cdot \frac{dt}{dx} = -\frac{2}{3t^2} \cdot \frac{1}{3t^2}$).

11. **(C)** $-\frac{\sqrt{y}}{\sqrt{x}}$

12. **(B)** $\frac{1}{2}$ ($y = \tan^{-1}(\tan\frac{x}{2}) = \frac{x}{2}$).

13. **(C)** $-\frac{\tan\sqrt{x}}{\sqrt{x}}$

14. **(D)** does not exist.

Assertion - Reason Questions (1 Mark)

15. **(D)** Assertion (A) is false, but Reason (R) is true. (For $k = \frac{5}{2}$, $2k = 5$, but $f(5) = 7$).

16. **(A)** Both (A) and (R) are true and (R) is the correct explanation.

17. **(A)** Both (A) and (R) are true and (R) is the correct explanation.

Very Short Answer Questions (2 Marks)

18. (a) Let $u = 2\cos^2 x$ and $v = \cos^2 x$. $\frac{du}{dv} = \frac{d(2v)}{dv} = \mathbf{2}$.

OR

(b) Differentiating implicitly: $\frac{1}{1+(x^2+y^2)^2} \cdot \left(2x + 2y\frac{dy}{dx}\right) = 0 \Rightarrow 2x + 2y\frac{dy}{dx} = 0 \Rightarrow \boldsymbol{\frac{dy}{dx} = -\frac{x}{y}}$.

19. (a) $x = e^{\frac{x}{y}} \Rightarrow \log x = \frac{x}{y} \Rightarrow y = \frac{x}{\log x}$. Differentiating: $\frac{dy}{dx} = \frac{\log x \cdot 1 - x \cdot \frac{1}{x}}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2}$. Substitute $\log x = \frac{x}{y}$ to get the required form.

OR

(b) LHD $= \lim_{h\to 0^-} \frac{f(-2+h)-f(-2)}{h} = 2$; RHD $= \lim_{h\to 0^+} \frac{f(-2+h)-f(-2)}{h} = 1$. Since LHD $\neq$ RHD, it is **not differentiable**.

20. (a) $\lim_{x\to -1} \frac{(x-3)(x+1)}{x+1} = \lim_{x\to -1}(x-3) = -4$. So $\mathbf{k = -4}$.

OR

(b) For $x \geq 0, f(x) = x^2 \Rightarrow f'(0) = 0$. For $x < 0, f(x) = -x^2 \Rightarrow f'(0) = 0$. It is **differentiable**.

21. (a) $y' = \frac{\sqrt{\cos x}(\cos x) - \sin x\left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos} x = \boldsymbol{\frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{\frac{3}{2}}}}$.

22. (a) $y = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = 5^x \log 5 \cdot x^{-5} + 5^x(-5x^{-6}) = \boldsymbol{\frac{5^{x}(x\log 5 - 5)}{x^6}}$.

Short Answer & Long Answer Questions (3 - 5 Marks)

26. (b) Simplify to $x^2(1+y) = y^2(1+x) \Rightarrow (x-y)(x+y+xy) = 0$. Given $x \neq y$, we have $y = -\frac{x}{1+x}$. Apply quotient rule to get $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$.

27. (b) Let $u = y^x, v = x^y, w = x^x$. Use logarithmic differentiation for each: $\frac{du}{dx} = y^x\left(\frac{x}{y}y' + \log y\right)$, $\frac{dv}{dx} = x^y\left(\frac{y}{x} + y'\log x\right)$, $\frac{dw}{dx} = x^x(1 + \log x)$. Sum and equate to 0.

28. (a) $y = e^{\sin x \log x} + e^{x \log \sin x}$. $\frac{dy}{dx} = x^{\sin x}\big(\frac{\sin x}{x} + \cos x \log x\big) + (\sin x)^{x(x \cot x + \log \sin x)}$.

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Section A (1 Mark)

1. **(C)** $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$.

2. **(C)** $-2\sqrt{\pi}$ ($y' = 2x\cos(x^2)$; at $x = \sqrt{\pi}, 2\sqrt{\pi}\cos\pi = -2\sqrt{\pi}$).

3. **(B)** 1 (Check $x = 3$: LHL $= -6$, RHL $= 20$. At $x = -3$: LHL $= 6$, RHL $= 6$).

4. **(B)** $\frac{2x}{1+x^4}$.

5. **(D)** continuous everywhere.

6. **(A)** $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$.

7. **(B)** $2e^x$ $\left(\frac{2e^{2x}}{e^x}\right)$.

8. **(B)** $\frac{1}{4}$ (Using L'Hopital's: $\lim_{x\to 0} \frac{1}{2\sqrt{4+x}} = \frac{1}{4}$).

9. **(A)** -1 ($e^y + xe^y y' = 0 \Rightarrow y' = -\frac{1}{x}$).

10. **(C)** $-2\cos x \cdot e^{\sin^2 x}$.

11. **(A)** $\frac{2}{3}$.

12. **(C)** 2 (Check at $x = 0$ and $x = 1$).

13. **(C)** -32 ($y' = f'\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) = \left(\frac{1}{x}\right)^3 \cdot \left(-\frac{1}{x^2}\right)$. At $x = \frac{1}{2}$, $y' = 8 \cdot (-4) = -32$).

14. **(A)** $\frac{1}{\pi}$.

15. **(B)** $-\frac{5}{3}\cot\theta$.

16. **(C)** 2.

Section B (2 Marks)

17. (a) $f(x) = x^3$ for $x \geq 0$ and $-x^3$ for $x < 0$. $f'(0) = \lim_{h\to 0} \frac{h^2|h|}{h} = 0$. **Differentiable**.

18. (a) At $x = \frac{\pi}{3}, \tan 2x$ is negative. $f(x) = -\tan 2x \Rightarrow f'(x) = -2\sec^2 2x$. At $\frac{\pi}{3}, -2(-2)^2 = \mathbf{-8}$.

19. (b) LHD $= \lim_{h\to 0^-} \frac{(1+h)^2+1-2}{h} = 2$; RHD $= \lim_{h\to 0^+} \frac{3-(1+h)-2}{h} = -1$. **Not differentiable**.

Section C (3 Marks)

24. (a) Substitute $x = \sin A, y = \sin B$. Equation becomes $\cos A + \cos B = a(\sin A - \sin B)$. Use transformation formulas: $2\cos\big(\frac{A+B}{2}\big)\cos\big(\frac{A-B}{2}\big) = a\big(2\cos\big(\frac{A+B}{2}\big)\sin\big(\frac{A-B}{2}\big)\big)$. $\cot\big(\frac{A-B}{2}\big) = a \Rightarrow A - B = 2\cot^{-1} a$. $\sin^{-1} x - \sin^{-1} y = \text{const}$. Differentiating gives the result.

25. (b) For continuity at $x = 2$: LHL $= \lim_{x\to 2^-}(-1 + a) = a - 1$. RHL $= \lim_{x\to 2^+}(1 + b) = b + 1$. $f(2) = a + b$. Solving $a - 1 = a + b \Rightarrow \boldsymbol{b = -1}$. Solving $b + 1 = a + b \Rightarrow \boldsymbol{a = 1}$.

6. APPLICATIONS OF DERIVATIVES

March, 2025

Multiple Choice Questions (1 Mark)

1. **(C)** 4 (Critical point $x = 1$; $f(0) = 2, f(1) = 0, f(2) = 4$).

2. **(D)** $[2, \infty)$ ($f'(x) = 2x - 4 \geq 0$).

3. **(C)** 1 cm/s ($100\pi = \pi(10)^2 \frac{dh}{dt}$).

4. **(C)** $\lambda \geq \sqrt{2}$ (Max of $\cos x + \sin x$ is $\sqrt{2}$).

5. **(C)** is an increasing function ($f'(x) = 2 - \sin x > 0$).

6. **(A)** (1, –10) (Slope $m = -3x^2 + 6x + 8$; m is max at $x = 1$).

7. **(C)** 375π ($V = \frac{4}{3}\pi\left(\frac{5}{4}(3x+1)\right)^3$; differentiate at $x = 1$).

8. **(B)** an increasing function.

9. **(B)** 2 sq units ($4\pi r^2 \frac{dr}{dt} = 2\frac{dr}{dt} \Rightarrow 4\pi r^2 = 2$).

10. **(D)** $e^{-\frac{1}{e}}$ (Min at $x = \frac{1}{e}$).

11. **(A)** – 6 ($f'(3) = 6 + a \geq 0$).

Very Short Answer Questions (2 Marks)

12. **Solution:** $f'(x) = \frac{15}{2}x^{\frac{1}{2}} - \frac{15}{2}x^{\frac{3}{2}} = \frac{15}{2}\sqrt{x}(1 - x)$.
    - (i) Increasing: **(0, 1)**
    - (ii) Decreasing: **(1, infinity)**.

13. **Solution:** $f'(x) = \cos x - a \geq 0 \Rightarrow a \leq \cos x$. Minimum $\cos x = -1$. So, $\boldsymbol{a \leq -1}$.

14. (a) **Solution:** $f'(x) = 4x - a \geq 0$ on $[2, 4]$. $f'(2) \geq 0 \Rightarrow 8 - a \geq 0 \Rightarrow a \leq 8$. Least value check: Usually implies the boundary of range. $\boldsymbol{a_{\max} = 8}$.

15. **Solution:** Slope $m = 5 - 6x^2$. $\frac{dm}{dt} = -12x\frac{dx}{dt}$. At $x = 2, \frac{dx}{dt} = 2 \Rightarrow -12(2)(2) = \mathbf{-48}$ **units/s²**.

16. **Solution:** $\frac{dS}{dt} = 8\pi r\frac{dr}{dt} = 5$. At $r = 8$, $\frac{dr}{dt} = \frac{5}{64\pi}$. $\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt} = 4\pi(64)\left(\frac{5}{64\pi}\right) =$ **20 mm³/s**.

17. **Solution:** $x^2 + y^2 = 13^2$. $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$. $12(2) + 5\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = \mathbf{-4.8}$ **m/s**.

18. **Solution:** $m = 7 - 3x^2$. $\frac{dm}{dt} = -6x\frac{dx}{dt} = -6(5)(2) = \mathbf{-60}$ **units/s²**.

19. **Solution:** $V = \pi r^2 h$. $\frac{dV}{dt} = \pi\left[2rh\frac{dr}{dt} + r^2\frac{dh}{dt}\right] = \pi[2(4)(7)(3) + 16(-5)] =$ $\boldsymbol{88\pi}$ **cm³/s**.

Short Answer & Long Answer Questions (3 - 5 Marks)

20. **Solution:** $A = \frac{\sqrt{3}}{4}s^2$. $\frac{dA}{dt} = \frac{\sqrt{3}}{2}s\frac{ds}{dt} = \frac{\sqrt{3}}{2}(15)(3) = \boldsymbol{\frac{45\sqrt{3}}{2}}$ **cm²/s**.

21. **Solution:** $f'(x) = 6(x-2)(x-3)$. Check $f(1) = 24, f(2) = 29, f(3) = 28, f(5) = 56$. **Abs Max = 56, Abs Min = 24**.

22. **Solution:** $f'(x) = \sqrt{3}\cos x + \sin x - 2a \leq 0 \Rightarrow 2a \geq 2 \Rightarrow \boldsymbol{a \geq 1}$.

23. **Solution:** $x, \frac{289}{x}$. Sum $S = x + \frac{289}{x}$. $S' = 0 \Rightarrow x^2 = 289 \Rightarrow x = 17$. Numbers are **17, 17**.

24. **Solution:** Maximize $m = -3x^2 + 6x + 9$. $m' = 0 \Rightarrow x = 1$. $m(1) = \mathbf{12}$.

25. **Solution:** $f'(x) = 12x(x-2)(x+1)$. Strictly Inc: $\boldsymbol{(-1, 0) \cup (2, \infty)}$; Strictly Dec: $\boldsymbol{(\infty, -1) \cup (0, 2)}$.

Case Study Based Questions (4 Marks)

29. (i) $2x + 3y = 300$. (ii) $A = x\left(100 - \frac{2}{3}x\right)$. (iii)(a) $x = 75, y = 50$. **Max Area = 3750 m²**.

30. (i) $S = 2x^2 + \frac{4V}{x}$. (ii) $\frac{dS}{dx} = 4x - \frac{4V}{x^2}$. (iii)(a) $4x^3 = 4V \Rightarrow \boldsymbol{x = y}$.

31. (i) $R(x) = (4000 + x)(10000 - 2x)$. (ii) $2000 - 4x$. (iii)(a) ₹500.

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Section A (1 Mark)

1. **(B)** $f'(x) > 0$.

2. **(B)** strictly increasing ($f' = 3(x-1)^2 + 9 > 0$).

3. **(B)** 6 cm/s ($P = 4s \Rightarrow \frac{dP}{dt} = 4(1.5)$).

4. **(A)** $k > 1$.

5. **(A)** 2.

6. **(A)** -60 units/sec.

7. **(D)** inflexion.

8. **(B)** π cm/s ($2\pi(0.5)$).

9. **(A)** Both A and R are true and R is correct explanation.

Section B (2 Marks)

11. **Solution:** $f(1) = 2(\text{min}), f(-1) = -2(\text{max})$. $M - m = -2 - 2 = \mathbf{-4}$.

12. **Solution:** $f' = 4x^2(x-3) < 0 \Rightarrow (\infty, \mathbf{3})$.

13. **Solution:** $V = a^3, \frac{da}{dt} = \frac{2}{a^2}$. $S = 6a^2 \Rightarrow \frac{dS}{dt} = \frac{24}{a} = \frac{24}{8} = \mathbf{3\ cm^2/s}$.

14. **Solution:** $\frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2$. $\frac{dC}{dt} = 2\pi \frac{dr}{dt} = \frac{2}{r} = \mathbf{0.4\ cm/s}$.

15. **Solution:** $f' = \frac{1-\log x}{x^2} = 0 \Rightarrow \boldsymbol{x = e}$.

Section D (5 Marks)

20. (a) **Solution:** $f'(1) = 0 \Rightarrow 4 - 124 + a = 0 \Rightarrow \boldsymbol{a = 120}$. Critical points: $\boldsymbol{x = 1, 5, -6}$. (b) **Solution:** $2(l + b) = 300 \Rightarrow l + b = 150$. Maximize $V = \pi r^2 h$. Dimensions: $\boldsymbol{l = 100\ \text{cm}, b = 50\ \text{cm}}$.

Section E (4 Marks)

21. (i) $\theta = \tan^{-1}\left(\frac{5}{x}\right)$. (ii) $-\frac{5}{x^2+25}$. (iii)(a) $\frac{d\theta}{dt} = -\frac{5}{2525} \times 20 = -\mathbf{\frac{4}{101}}$ **rad/s**.

22. (i) 7.2. (iii)(a) 62.5 km/h.

23. (i) $x = 450$. (ii) Rebate $= 350 - 225 =$ **₹125**.

24. (i) $A = (x+3)\left(\frac{24}{x} + 2\right)$. (ii) $x = 6, y = 4$. Card: **9 cm × 6 cm**.

25. (i) $x = y\sqrt{3}$. (ii) $t \approx 2.26$ s. (iii)(b) **19.6 m/s** ($v = 9.8 \times 2$).

7. INTEGRALS

March, 2025

Multiple Choice Questions (1 Mark)

1. **(B)** 0 (The integrand $f(x) = |x\frac{1}{x}$ is an odd function).

2. **(A)** $-\frac{1}{\log}2$ (Put $\frac{1}{x} = t \Rightarrow -\frac{1}{x^2}dx = dt$).

3. **(D)** $2\int_0^a f(x)\,dx$ (Property of definite integrals).

4. **(A)** $2(\sin x + x\cos\alpha) + C$ (Use $\cos 2\theta = 2\cos^2\theta - 1$).

5. **(C)** $\tan^{-1} e - \frac{\pi}{4}$ (Put $e^x = t$).

6. **(B)** $2(\sin\frac{x}{2} - \cos\frac{x}{2}) + C$ (Write $1 + \sin x = (\sin\frac{x}{2} + \cos\frac{x}{2})^2$).

7. **(C)** $e - 1$ (Put $\sin x = t$).

8. **(C)** $\frac{x^4}{4}$ + C (Integrand simplifies to x^3).

9. **(A)** $\int_a^b f(x)$ dx $= \int_a^b f(a+b-x)$ dx (King's Property).

10. **(D)** $\sin^{-1}(\frac{e^x}{2})$ + C (Put $e^x = t$).

11. **(D)** $6\sin\sqrt{x} + C$ (Put $\sqrt{x} = t$).

12. **(A)** $\frac{1}{4}\tan^{-1}\frac{x^4}{2}$ + C (Put $x^4 = t$).

13. **(B)** $e^x \cdot \frac{1}{1+x}$ + C (Use form $\int e^x[f(x) + f'(x)]dx$).

14. **(D)** $\frac{1}{9}\tan^{-1}\frac{3x+1}{3} + C$ (Complete the square: $(3x+1)^2 + 3^2$).

15. **(B)** 0 (Integrand is an odd function).

Very Short Answer Questions (2 Marks)

16. **Solution:** $\int_0^{\frac{\pi}{4}} \sqrt{(\sin x + \cos x)^2}dx = \int_0^{\frac{\pi}{4}}(\sin x + \cos x)dx = [-\cos x + \sin x]_0^{\frac{\pi}{4}} = \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (-1 + 0) = \mathbf{1}$.

Short Answer & Long Answer Questions (3 - 5 Marks)

17. (a) **Solution:** $\int \frac{x+2\sin(\frac{x}{2})\cos(\frac{x}{2})}{2\cos^2(\frac{x}{2})}dx = \int\left[\frac{1}{2}x\sec^2(\frac{x}{2}) + \tan(\frac{x}{2})\right]dx$ This is of the form $\int[xf'(x) + f(x)]dx = xf(x)$. Result: $\boldsymbol{x\tan(\frac{x}{2}) + C}$.

20. **Solution:** $\int e^x \frac{1-2\sin(\frac{x}{2})\cos(\frac{x}{2})}{2\sin^2(\frac{x}{2})}dx = \int e^x\left[\frac{1}{2}\csc^2(\frac{x}{2}) - \cot(\frac{x}{2})\right]dx$ Use $\int e^x[f(x) + f'(x)]dx = e^x f(x)$. Result: $\boldsymbol{-e^x\cot(\frac{x}{2})|_{\frac{\pi}{2}}} = \boldsymbol{e^{\frac{\pi}{2}}}$.

22. (b) **Solution:** Split the integral at the roots of the modulus: $\int_1^2[(2-x) + (4-x)]dx + \int_2^4[(x-2) + (4-x)]dx = \int_1^2(6-2x)dx + \int_2^4(2)dx = \left[6x - x^2\right]_1^2 + [2x]_2^4 = (12-4) - (6-1) + (8-4) = 3 + 4 = \mathbf{7}$.

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Section A: Multiple Choice Questions (1 Mark)

1. **(B)** $\int_a^b f(a+b-x)dx$.

2. **(B)** Zero (0) (Apply $x \to \frac{\pi}{2} - x$ and add the two integrals).

3. **(B)** f(– x) = – f(x) (Property of odd functions).

4. **(C)** $\frac{\pi}{2}$ (Standard formula $\sin^{-1}(\frac{x}{3})$).

5. **(B)** $-\frac{1}{\log}x + c$ (Put $\log x = t$).

6. **(D)** 0 (Odd function on symmetric interval).

7. **(D)** $-\cot x - \tan x + c$ (Write $\cos 2x = \cos^2 x - \sin^2 x$).

8. **(A)** 0 (Odd function).

9. **(D)** $\frac{e^x}{(x-1)^2} + C$ (Write $x - 3 = (x-1) - 2$ and use $e^{x[f+f']}$).

Section B: Very Short Answer Type Questions (2 Marks)

10. (b) **Solution:** Put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$. Limits: 0 to $\frac{\pi}{2}$. $\int_0^{\frac{\pi}{2}} 2\sin t dt = [-2\cos t]_0^{\frac{\pi}{2}} = 0 - (-2) = \mathbf{2}$.

16. (b) **Solution:** Roots of $x + 2 = 0$ is $x = -2$. $\int_{-4}^{-2} -(x+2)dx + \int_{-2}^{0}(x+2)dx = \left[-\frac{x^2}{2} - 2x\right]_{-4}^{-2} + \left[\frac{x^2}{2} + 2x\right]_{-2}^{0} = 2 + 2 = \mathbf{4}$.

Section C: Short Answer Type Questions (3 Marks)

21. (a) **Solution:** $\int e^x \frac{2+2\sin x\cos x}{2\cos^2 x}dx = \int e^x(\sec^2 x + \tan x)dx$. Using $\int e^x[f(x) + f'(x)]dx$, result is $\boldsymbol{e^x \tan x + C}$.

25. **Solution:** Put $x^2 = t \Rightarrow 2xdx = dt$. $\int \frac{1}{(t+1)(t+3)}dt = \frac{1}{2}\int\left[\frac{1}{t+1} - \frac{1}{t+3}\right]dt = \frac{1}{2}\log\left|\frac{t+1}{t+3}\right| + C = \boldsymbol{\frac{1}{2}\log\left|\frac{x^2+1}{x^2+3}\right| + C}$.

Section D: Long Answer Type Questions (5 Marks)

28. (b) **Solution:** Put $\sin x = t \Rightarrow \cos x dx = dt$. $2\sin x\cos x = 2t \cdot dt$. Limits 0 to 1. $I = \int_0^1 2t\tan^{-1} t dt$. Integrating by parts: $t^2\tan^{-1} t - \int \frac{t^2}{1+t^2}dt = t^2\tan^{-1} t - \int\left(1 - \frac{1}{1+t^2}\right)dt = \left[t^2\tan^{-1} t - t + \tan^{-1} t\right]_0^1 = \left(\frac{\pi}{4} - 1 + \frac{\pi}{4}\right) = \boldsymbol{\frac{\pi}{2} - 1}$.

8. APPLICATIONS OF INTEGRALS

March, 2025

Multiple Choice Questions (1 Mark)

1. **(D)** $\int_0^4 \sqrt{x}dx$

2. **(D)** $\int_0^4 \sqrt{y}\,dy$

3. **(C)** $\frac{16}{3}$ sq. units

4. **(B)** $\frac{16}{3}$

5. **(C)** $\frac{4}{3}$ sq units

6. **(A)** 6 sq units

7. **(B)** π sq. units

Very Short Answer Questions (2 Marks)

8. **Solution:** The curve is an ellipse $\frac{x^2}{3^2}+\frac{y^2}{2^2}=1$. The area bounded by the curve and the x-axis (upper half) is: Area $=\int_{-3}^{3} y\,dx = \int_{-3}^{3}\frac{2}{3}\sqrt{9-x^2}\,dx = \frac{2}{3}\times 2\int_0^3 \sqrt{9-x^2}\,dx = \frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\frac{x}{3}\right]_0^3 = \frac{4}{3}\left[0+\frac{9}{2}\cdot\frac{\pi}{2}\right] = \mathbf{3\pi}$ **sq. units**.

Short Answer & Long Answer Questions (3 - 5 Marks)

9. **Solution:** $y=|x+3|$ breaks at $x=-3$. Area $=\int_{-6}^{-3}-(x+3)dx+\int_{-3}^{0}(x+3)dx = \left[-\frac{x^2}{2}-3x\right]_{-6}^{-3}+\left[\frac{x^2}{2}+3x\right]_{-3}^{0} = \mathbf{9}$ **sq. units**.

10. **Solution:** Line $y=5x+2$ intersects x-axis at $x=-\frac{2}{5}$. Area $=|\int_{-2}^{-\frac{2}{5}}(5x+2)dx|+\int_{-\frac{2}{5}}^{2}(5x+2)dx = \mathbf{20.8}$ **sq. units**.

11. **Solution:** The region is in the first quadrant bounded by $x=0$, $y=9$, and $y=x^2$. Area $=\int_0^3(9-x^2)dx = \left[9x-\frac{x^3}{3}\right]_0^3 = 27-9 = \mathbf{18}$ **sq. units**.

12. **Solution:** Circle $x^2+y^2=64$ and line $y=x$ (since $\theta=\frac{\pi}{4}$). Area $=\int_0^{\frac{8}{\sqrt{2}}} x\,dx + \int_{\frac{8}{\sqrt{2}}}^{8}\sqrt{64-x^2}\,dx = \mathbf{8\pi}$ **sq. units**.

13. **Solution:** Intersection of $y=x^2$ and $y=x$ is $(0,0)$ and $(1,1)$. Area $=\int_0^1 x^2dx+\int_1^3 xdx = \frac{1}{3}+(4.5-0.5) = \mathbf{\frac{13}{3}}$ **sq. units**.

14. **Solution:** Total Area $=2\int_0^4\sqrt{x}dx=\frac{32}{3}$. Area from 0 to a: $2\int_0^a\sqrt{x}dx=\frac{4}{3}a^{\frac{3}{2}}$. Set $\frac{4}{3}a^{\frac{3}{2}}=\frac{1}{2}\left(\frac{32}{3}\right)\Rightarrow a^{\frac{3}{2}}=4\Rightarrow \boldsymbol{a=4^{\frac{2}{3}}}$.

15. **Solution:** Area $=2\int_{-2}^{2}\sqrt{16-x^2}dx = \mathbf{8\sqrt{3}+\frac{16\pi}{3}}$ **sq. units**.

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Section A (1 Mark)

1. **(D)** $\frac{4}{3}$

2. **(C)** 2

Section D (5 Marks)

3. **Solution:** $y=\frac{1}{2}\sqrt{16-x^2}$. Area result: $\mathbf{4\sqrt{3}+\frac{8\pi}{3}}$ **sq. units**.

4. **Solution:** $A_1=\int_0^1 2\sqrt{x}dx=\frac{4}{3}$. $A_2=\int_0^4 2\sqrt{x}dx=\frac{32}{3}$. Ratio $A_1:A_2=\mathbf{1:8}$.

5. **Solution:** (Same as Question 15, March 2025): $\mathbf{8\sqrt{3}+\frac{16\pi}{3}}$ **sq. units**.

6. (a) **Solution:** $y=-x^2$ for $x<0$ and x^2 for $x>0$. Area $=\int_{-2}^{0}|-x^2|\,dx+\int_0^2 x^2dx = \mathbf{\frac{16}{3}}$ **sq. units**.

OR

(b) **Solution:** $\frac{x^2}{25}+\frac{y^2}{9}=1$. Area $=\frac{12\sqrt{21}}{5}+30\sin^{-1}\left(\frac{2}{5}\right)$.

7. **Solution:** Ellipse $\frac{x^2}{9}+\frac{y^2}{36}=1$. $a=3, b=6$. Area $=\pi ab=\mathbf{18\pi}$ **sq. units**.

8. **Solution:** Area $=\int_{-1}^{1}x^2dx=\mathbf{\frac{2}{3}}$ **sq. units**.

9. (a) **Solution:** Area $=\int_{-\sqrt{2}}^{\sqrt{3}}\sqrt{4-x^2}dx = \mathbf{\frac{\sqrt{3}}{2}+\frac{7\pi}{6}+1}$ **sq. units**.

OR

(b) **Solution:** Area $= 2\int_1^3 \sqrt{y}dy = \mathbf{4\sqrt{3} - \frac{4}{3}}$ **sq. units**.

9. DIFFERENTIAL EQUATIONS

March, 2025

Multiple Choice Questions (1 Mark)

1. **(B)** $\frac{1}{\sqrt{y}}$
2. **(B)** 1 (Equation: $3(y')^2 y'' = 0 \Rightarrow$ order $p = 2$, degree $q = 1$)
3. **(C)** order 2, degree not defined (due to $\sin\left(d\frac{y}{d}x\right)$)
4. **(C)** $e^{2\sqrt{x}}$
5. **(C)** 3 (Order 2, Degree 1)
6. **(B)** $e^{x^2} + e^{-y} = C$
7. **(B)** 3 (Order 2, Degree 1)
8. **(B)** $\frac{1}{x}$
9. **(D)** $\tan^{-1} y = \tan^{-1} x + C$
10. **(C)** $\sin x$

Assertion - Reason Questions (1 Mark)

11. **(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation.

Short Answer & Long Answer Questions (3 - 5 Marks)

12. (a) **Solution:** $\int \frac{y}{y+3} dy = \int \frac{2}{x} dx \Rightarrow y - 3\log|y+3| = 2\log|x| + C$. Using $y(1) = -2 \Rightarrow C = -2$. **Particular Solution:** $y - 3\log|y+3| = 2\log|x| - 2$.

OR

(b) **Solution:** Linear form: $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$. IF $= 1 + x^2$. Solution: $y(1+x^2) = \frac{4x^3}{3} + C$.

13. **Solution:** Homogeneous. Put $y = vx$. Result: $\cot\left(\frac{y}{x}\right) = \log|x| + 1$.

17. (b) **Solution:** $\frac{d}{dx}(xy) = x\cos x + \sin x$. Integrating: $xy = x\sin x + C$. $y\left(\frac{\pi}{2}\right) = 1 \Rightarrow C = 0$. **Solution:** $y = \sin x$.

Case Study Based Questions (4 Marks)

18. **Solution:** (i) $T(t) = 25 + 60e^{-kt}$. (ii) $t = \frac{1.3863}{0.03} = \mathbf{46.21}$ **minutes**.
19. **Solution:** (i) Order 1, Degree 1. (ii) $r = \frac{2}{3}kt + 5$. (iii) (a) $\boldsymbol{k = -3}$. For $r = 0$, $\boldsymbol{t = 2.5}$ **hours**.

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Section A (1 Mark)

1. **(D)** $\frac{1}{\sqrt{1-x^2}}$
2. **(C)** 2, 1
3. **(A)** $\cos x - \sin\left(\frac{y}{x}\right)$
4. **(D)** not defined
5. **(C)** first order linear differential equation
6. **(A)** $xy = c$
7. **(D)** $\frac{1}{y}$
8. **(A)** 4
9. **(A)** $\log|1+y| = x - \frac{x^2}{2} + c$
10. **(B)** 3
11. **(C)** $\sec x$
12. **(B)** 1

Section C (3 Marks)

13. **Solution:** Put $y = vx$. Result: $2\tan\left(\frac{y}{2x}\right) = \log|x| + 2$.
14. (b) **Solution:** $d\frac{x}{d}y - \frac{1}{y}x = 2y$. IF $= \frac{1}{y}$. Solution: $\boldsymbol{x = 2y^2 + Cy}$.
16. (a) **Solution:** IF $= e^{-x^2}$. Particular Solution: $y = (x^3 + 5)e^{x^2}$.

18. Solution: $y = -\frac{x(\log x)^2}{2} + Cx$.

Section E (4 Marks)

19. Solution: (i) $\boldsymbol{P = Ce^{kt}}$. (ii) $\boldsymbol{k = \log_e 2}$.

10. VECTOR ALGEBRA

March, 2025

Multiple Choice Questions (1 Mark)

1. (B) $(a) \perp (b)$

2. (C) $\frac{\pi}{3}$

3. (A) $\frac{(a)\cdot(b)}{|(b)|^2}(b)$

4. (D) $\frac{2\pi}{3}$

5. (C) $\frac{\sqrt{34}}{2}$ units

6. (C) (– 2, 2)

7. (C) 24 and 8

8. (D) 4

9. (A) orthogonal vectors

10. (B) 3

11. (C) 45°

12. (B) two

13. (C) $\pm\frac{1}{\sqrt{3}}$

Assertion - Reason Questions (1 Mark)

14. (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation.

Very Short Answer Questions (2 Marks)

15. Solution: Area = $\frac{\sqrt{62}}{2}$ **sq. units**.

16. (a) **Solution:** $\theta = \cos^{-1}\left(\frac{\sqrt{21}}{7}\right)$.

OR

(b) **Solution:** Opposite direction vector = $-18\hat{\imath} + 6\hat{\jmath} + 9\hat{k}$.

17. Solution: $x = \frac{1}{4}$.

18. (b) **Solution:** Area = **16 sq. units**.

19. (a) **Solution:** $\Sigma\vec{a}\cdot\vec{b} = -\frac{29}{2}$.

Short Answer & Long Answer Questions (3 - 5 Marks)

24. (a) **Solution:** $\theta = 60^{\circ}$.

25. Solution: Area = $\frac{\sqrt{61}}{2}$ **sq. units**.

Case Study Based Questions (4 Marks)

29. Solution: (ii) $\overrightarrow{AC} = 4\vec{a} - 2\vec{b}$, $\overrightarrow{BC} = 5\vec{a} - 3\vec{b}$.
(iii) (a) $\theta = 60^{\circ}$, $|\vec{a}\times\vec{b}| = \sqrt{3}$.

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Section A (1 Mark)

1. (C) $\pm\frac{4}{5}$

2. (D) $-\hat{\imath} + \hat{\jmath} - 2\hat{k}$

3. (C) $\vec{a}\cdot\vec{b} \leq |\vec{a}|\ |\vec{b}|$

4. (B) $\hat{\jmath}$

5. (C) perpendicular vectors

6. (D) [0, 6]

7. (C) $\frac{5\pi}{6}$

8. (D) a right-angled triangle

9. (B) $2a^2$

10. (D) $\frac{5\vec{p}+3\vec{q}}{8}$

11. (C) $\frac{\pi}{6}$

12. (B) 5

13. (D) $-\frac{3}{2}$

14. (A) $\frac{3}{7}$

15. (A) $\frac{\pi}{4}$

16. (C) Assertion is true, but Reason is false.

17. (D) Assertion is false, but Reason is true.

18. (A) Both true and Reason explains Assertion.

Section B (2 Marks)

19. Solution: $\vec{C} = \frac{-5\hat{\imath}+2\hat{\jmath}+5\hat{k}}{3}$. Ratio is **3 : 1**.

20. Solution: Equality holds when $\theta = \mathbf{90}^{\circ}$.

Section C (3 Marks)

22. Solution: Area $= \mathbf{11\sqrt{5}}$ **sq. units**.

23. Solution: Required vector $= \mathbf{2\sqrt{2}}\left(\hat{\jmath} + \hat{k}\right)$.

Section E (4 Marks)

26. Solution: (i) $|\overrightarrow{AV}| = \sqrt{\mathbf{113}}$. (ii) Unit vector $= \frac{5\hat{\imath}+2\hat{\jmath}+4\hat{k}}{\sqrt{45}}$. (iii) Projection $= \frac{11}{3\sqrt{5}}$.

11. 3D GEOMETRY

March, 2025

Multiple Choice Questions (1 Mark)

1. **(B)** $\frac{\pi}{3}$ only
2. **(C)** $x = 3t + 4, y = t - 3, z = 2t + 7$.
3. **(C)** $(1, -5, -6)$.
4. **(A)** $-\frac{3}{2}$

Assertion - Reason Questions (1 Mark)

5. **(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation.

Very Short Answer Questions (2 Marks)

6. **Solution:** P: $\left(\mathbf{\frac{14}{3}}, \mathbf{\frac{4}{3}}, -\mathbf{\frac{7}{3}}\right)$. Q: $\left(\mathbf{\frac{16}{3}}, \mathbf{\frac{5}{3}}, -\mathbf{\frac{8}{3}}\right)$.
7. **Solution:** $\boldsymbol{\theta} = \mathbf{cos}^{-1}\left(\mathbf{\frac{11}{7\sqrt{3}}}\right)$.
8. **Solution:** $b_1 \cdot b_2 = 0 \Rightarrow \boldsymbol{\theta} = \mathbf{90}^{\circ}$.
9. **Solution:** D.R.s: **(4, −3, 0)**. Point: **(2, 2.5, −1)**.

Short Answer & Long Answer Questions (3 - 5 Marks)

10. (b) **Solution:** $k = \frac{2}{3}$. Ratio is **2 : 3**.
11. (a) **Solution:** Image A': $(\mathbf{1}, \mathbf{0}, \mathbf{7})$. Line: $\mathbf{\frac{x-1}{0}} = \mathbf{\frac{y-6}{-6}} = \mathbf{\frac{z-3}{4}}$.
12. **Solution:** Intersection point: **(-1, −1, −1)**. Distance: **9.85**.
13. (a) **Solution:** Foot: **(0.5, 0, 5.5)**.
14. (a) **Solution:** Distance: $\mathbf{\frac{\sqrt{293}}{7}}$.

Case Study Based Questions (4 Marks)

21. (i) **No**. (ii) $\mathbf{\frac{x-1}{3}} = \mathbf{\frac{y+2}{-2}} = \mathbf{\frac{z+3}{4}}$. (iii)(a) Line: $\vec{\boldsymbol{r}} = (\mathbf{3}, \mathbf{2}, \mathbf{1}) + \boldsymbol{k}(\mathbf{2}, \mathbf{17}, \mathbf{7})$.

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Section A (1 Mark)

1. **(D)** $\pm\frac{1}{3}$
2. **(C)** $\sqrt{a^2 + c^2}$.
3. **(D)** (0, 0, 0).
4. **(D)** 2, − 1, 3
5. **(A)** 90°
6. **(D)** $\cos\alpha + \cos\beta + \cos\gamma = 1$.
7. **(C)** $\frac{\pi}{2}$.
8. **(B)** $\vec{r} = \hat{\imath} - \hat{\jmath} + \lambda\hat{\jmath}$.
9. **(C)** 2
10. **(B)** $\frac{3\pi}{4}$
11. **(D)** $\frac{x-1}{1} = \frac{y+3}{1} = \frac{z-2}{2}$.
12. **(B)** $\frac{\pi}{2}$
13. **(C)** 1, 2, 3.
14. **(C)** 9, −3, 2
15. **(D)** $\frac{x-1}{1} = \frac{y+3}{1} = \frac{2-z}{-2}$.
16. **(D)** Assertion (A) is false, Reason (R) is true.

Section B (2 Marks)

17. **Solution:** $\vec{\boldsymbol{r}} = \left(\mathbf{2}\hat{\boldsymbol{\imath}} + \mathbf{3}\hat{\boldsymbol{\jmath}} - \mathbf{5}\hat{\boldsymbol{k}}\right) + \boldsymbol{\lambda}\left(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + \hat{\boldsymbol{k}}\right)$.
18. **Solution:** $\mathbf{90}^{\circ}$.

Section D (5 Marks)

19. **Solution:** P: $(\mathbf{1}, \mathbf{6}, \mathbf{3})$.
20. (a) **Solution:** Eq: $\mathbf{\frac{x-1}{13}} = \mathbf{\frac{y-3}{-2}} = \mathbf{\frac{z-5}{-3}}$.
21. (b) **Solution:** Foot: **(3, 5, 7)**. Image: **(1, 1, 11)**. Length: $\sqrt{\mathbf{29}}$.

22. (b) **Solution:** Intersection: $\mathbf{(1,\ -1,\ 6)}$. Eq:
$\vec{\boldsymbol{r}} = \left(\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}} + \mathbf{6}\hat{\boldsymbol{k}}\right) + \boldsymbol{k}\left(\hat{\boldsymbol{\imath}} + \hat{\boldsymbol{\jmath}} + \mathbf{3}\hat{\boldsymbol{k}}\right)$.

12. LINEAR PROGRAMMING

March, 2025

Multiple Choice Questions (1 Mark)

1. **(C)** Z is maximum at (40, 15), minimum at (15, 20)
2. **(C)** It will have both maximum and minimum values.
3. **(C)** The objective function maximizes the combined profit earned from selling X and Y.
4. **(D)** $3a = 2b$
5. **(B)** AOEC
6. **(B)** $a = \frac{b}{2}$
7. **(B)** 4
8. **(C)** at the corner points of the feasible region
9. **(D)** I Quadrant
10. **(A)** Open half-plane containing the origin.
11. **(C)** at the corners of the feasible region

Assertion - Reason Questions (1 Mark)

12. **(D)** Assertion (A) is false but Reason (R) is true.
13. **(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation.
14. **(A)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation.

Very Short Answer Questions (2 Marks)

15. **Solution:** Corner Points: $O(0,0), A(1,0), B(1,3), C(0,6)$.

Short Answer & Long Answer Questions (3 - 5 Marks)

16. **Solution:** Maximize $Z = 6$ at $(2,2)$.
17. **Solution:** Minimum $Z = -17$ at $(3,4)$.
18. **Solution:** Maximum $Z = 600$ at all points on the line segment joining $(60,30)$ and $(120,0)$.
19. **Solution:** Minimum $Z = 134$ at $(3,8)$.
20. **Solution:** Condition: $b = 3a$.

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Section A (1 Mark)

1. **(B)** linear function
2. **(C)** 2
3. **(D)** a feasible region
4. **(B)** constraints
5. **(D)** $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$
6. **(D)** 170
7. **(C)** open half-plane containing origin
8. **(D)** 20
9. **(B)** Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation.

Section C (3 Marks)

10. **Solution:** Max $Z = 3200$ at $(4,4)$.
11. **Solution:** Max $Z = 15$ at $(3,3)$.
12. **Solution:** Maximum: 300 at $(60,20)$. Minimum: 120 at $(0,40)$.

Section D (5 Marks)

14. **Solution:** Maximize $Z = 400$ at $(4,4)$.
15. **Solution:** Minimum $Z = 620$ at $(80,20)$.

PROBABILITY

March, 2025

Multiple Choice Questions (1 Mark)

1. **(C)** $\frac{2}{3}$
2. **(D)** $\frac{1-P(E\cup F)}{P(\overline{F})}$
3. **(D)** 0.7
4. **(D)** $\frac{1}{5}$
5. **(B)** $\frac{3}{26}$
6. **(A)** $\frac{1}{2}$
7. **(C)** $\frac{8}{15}$
8. **(D)** $\frac{7}{12}$

Assertion - Reason Questions (1 Mark)

9. **(D)** Assertion (A) is false, but Reason (R) is true.

Very Short Answer Questions (2 Marks)

10. (a) Probability distribution of X:

x	0	1	2	3
$P(X=x)$	2/10	3/10	4/10	1/10

Mean $= 0\times\frac{2}{10}+1\times\frac{3}{10}+2\times\frac{4}{10}+3\times\frac{1}{10} = 1.4$

OR

(b) Let A: person is a woman, B: person works outside the village.
$P(A)=\frac{4000}{8000}=0.5$, $P(B)=\frac{3000}{8000}=0.375$, $P(A\cap B)=\frac{1200}{8000}=0.15$.
$P(A\cup B)=P(A)+P(B)-P(A\cap B)=0.5+0.375-0.15=0.725$.

Short Answer & Long Answer Questions (3 - 5 Marks)

11. (a) Number of times 2 appears in two throws follows binomial distribution with $n=2$, $p=\frac{3}{10}$. Mean $=np=2\times\frac{3}{10}=0.6$.

OR

(b) $P(A)=\frac{4}{36}=\frac{1}{9}$, $P(B)=\frac{30}{36}=\frac{5}{6}$, $P(A\cap B)=\frac{3}{36}=\frac{1}{12}$.
Since $P(A\cap B)\neq 0$, events are not mutually exclusive.
$P(A)\cdot P(B)=\frac{1}{9}\times\frac{5}{6}=\frac{5}{54}\neq\frac{1}{12}$, so events are not independent.

12. (a) Probability distribution of number of boys (X) in three children:

X	0	1	2	3
$P(X)$	1/8	3/8	3/8	1/8

OR

(b) Probability distribution of $X=$ (number of heads − number of tails) in two tosses:

X	−2	0	2
$P(X)$	1/4	1/2	1/4

Mean $=(-2)\times\frac{1}{4}+0\times\frac{1}{2}+2\times\frac{1}{4}=0$.

13. (a) Let B: buys coloring book, C: buys box of colors.
Given: $P(B)=0.7$, $P(C)=0.2$, $P(B|C)=0.3$.
(i) $P(B\cap C)=P(B|C)\cdot P(C)=0.3\times 0.2=0.06$.
(ii) $P(C|B)=\frac{P(B\cap C)}{P(B)}=\frac{0.06}{0.7}=\frac{3}{35}$.

OR

(b) Number of oranges drawn in three trials (with replacement) follows binomial distribution with $n=3$, $p=\frac{4}{10}=0.4$.
(i) Probability distribution:

k	0	1	2	3
$P(X=k)$	0.216	0.432	0.288	0.064

(ii) Expectation $=np=3\times 0.4=1.2$.

14. (a) Number of doublets in three throws of a pair of dice follows binomial distribution with $n=3$, $p=\frac{1}{6}$.

X	0	1	2	3
$P(X)$	125/216	75/216	15/216	1/216

OR

(b) Given $P(E) = p$, $P(F) = 2p$, independent, and $P(\text{exactly one}) = \frac{5}{9}$.
$P(\text{exactly one}) = p(1-2p) + (1-p)(2p) = 3p - 4p^2 = \frac{5}{9}$.
Solving: $36p^2 - 27p + 5 = 0$ gives $p = \frac{1}{3}$ or $p = \frac{5}{12}$.

15. (a) Box: 3 black, 7 red balls. Two balls drawn without replacement.
(i) $P(\text{both red}) = \frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$.
(ii) $P(\text{first black, second red}) = \frac{3}{10} \times \frac{7}{9} = \frac{7}{30}$.

OR

(b) Number of face cards in two draws (with replacement) follows binomial with $n = 2$, $p = \frac{12}{52} = \frac{3}{13}$.

X	0	1	2
$P(X)$	100/169	60/169	9/169

Case Study Based Questions (4 Marks)

16. (i) $P(\text{default}) = 0.1 \times 0.05 + 0.2 \times 0.03 + 0.7 \times 0.01 = 0.018$.
(ii) $P(\text{variable rate} \mid \text{default}) = \frac{0.7\times0.01}{0.018} = \frac{7}{18}$.

17. (i) (a) $P(\text{electric}) = 0.6 \times 0.2 + 0.3 \times 0.1 + 0.1 \times 0.05 = 0.155$.

OR

(i) (b) $P(\text{petrol}) = 1 - 0.155 = 0.845$.
(ii) $P(\text{Comet} \mid \text{electric}) = \frac{0.1\times0.05}{0.155} = \frac{1}{31}$.
(iii) $P(\text{Amber or Bonzi} \mid \text{electric}) = 1 - \frac{1}{31} = \frac{30}{31}$.

18. (i) $P(\text{no misconception and answers Bijoy}) = 0.6 \times 0.1 = 0.06$.
(ii) $P(\text{answers Bijoy}) = 0.4 \times 0.8 + 0.6 \times 0.1 = 0.38$.
(iii) (a) $P(\text{misconception} \mid \text{answers Bijoy}) = \frac{0.4\times0.8}{0.38} = \frac{16}{19}$.

OR

(iii)
(b) $P(\text{no misconception} \mid \text{answers Bijoy}) = 1 - \frac{16}{19} = \frac{3}{19}$.

19. (i) $P(\text{germinate}) = \frac{10}{30} \times 0.25 + \frac{12}{30} \times 0.35 + \frac{8}{30} \times 0.40 = 0.33$.
(ii) $P(\text{cabbage} \mid \text{germinates}) = \frac{\frac{12}{30}\times0.35}{0.33} = \frac{14}{33}$.

20. (i) $P(\text{contracted}) = 0.7 \times 0.25 + 0.2 \times 0.35 + 0.1 \times 0.50 = 0.295$.
(ii) $P(A_2 \mid \text{not contracted}) = \frac{0.2\times0.65}{1-0.295} = \frac{26}{141}$.

21. Assuming $P(\text{first appointed}) = 0.5$, $P(\text{second appointed}) = 0.4$, and mutually exclusive events (with $P(\text{neither}) = 0.1$):
(i) $P(\text{plant introduced}) = 0.5 \times 0.7 + 0.4 \times 0.4 = 0.51$.
(ii) $P(\text{first appointed} \mid \text{plant introduced}) = \frac{0.5\times0.7}{0.51} = \frac{35}{51}$.

22. (i) $P(\text{unable to get good marks}) = 0.1 \times 0.002 + \frac{13}{30} \times 0.02 + \frac{7}{15} \times 0.2 = 0.1022$.
(ii) $P(\text{not category A} \mid \text{unable to get good marks}) = 1 - \frac{0.1\times0.002}{0.1022} \approx 0.998$.

23. (i) $P(\text{exactly two hit}) = \frac{13}{30}$.
(ii) $P(\text{at least one hits}) = 1 - \frac{1}{60} = \frac{59}{60}$.

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Section A: Multiple Choice Questions (1 Mark)

1. **(C)** $P(A \cap B) = \frac{1}{2}P(B)$
2. **(C)** 1
3. **(D)** 0
4. **(D)** $P(A) = P(B)$
5. **(B)** $\frac{5}{9}$

Section A: Assertion-Reason (1 Mark)

6. **(C)** Assertion (A) is true, but Reason (R) is false.

Section C: Short Answer Type Questions (3 Marks)

7. Given $P(\overline{E}) = 0.6 \Rightarrow P(E) = 0.4$, and $P(E \cup F) = 0.6$ for independent events.
$0.6 = 0.4 + P(F) - 0.4 \cdot P(F) \Rightarrow 0.2 = 0.6P(F) \Rightarrow P(F) = \frac{1}{3}$.
$P(\overline{E} \cup \overline{F}) = 1 - P(E \cap F) = 1 - 0.4 \times \frac{1}{3} = \frac{13}{15}$.

8. Probability distribution of absolute difference X:

X	0	1	2	3	4	5
$P(X)$	1/6	5/18	2/9	1/6	1/9	1/18

9. $P(\text{R} \mid \text{increased profits}) = \frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5} = \frac{1}{3}$.

10. (a) $P(\text{lost card is King} \mid \text{drawn card is King}) = \frac{1}{17}$.

OR

(b) Biased die: $P(\text{six}) = \frac{2}{9}$. Distribution of number of sixes in two throws:

X	0	1	2
$P(X)$	49/81	28/81	4/81

Mean $= \frac{4}{9}$.

11. (a) $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{13}$, $P(A \cap B) = \frac{1}{52}$. Since $P(A) \cdot P(B) = P(A \cap B)$, events are independent.

OR

(b) Probability distribution of number of doublets in three throws:

X	0	1	2	3
$P(X)$	125/216	75/216	15/216	1/216

12. $P(\text{irregular} \mid \text{A grade}) = \frac{0.8\times 0.2}{0.2\times 0.8+0.8\times 0.2} = 0.5$.

Section E: Case Study Based Questions (4 Marks)

13. (i) $P(\text{late}) = \frac{1}{3} \times 0.55 + \frac{1}{3} \times 0.37 + \frac{1}{3} \times 0.17 = \frac{1.09}{3} = \frac{109}{300}$.
(ii) $P(\text{moderate} \mid \text{late}) = \frac{0.37}{1.09} = \frac{37}{109}$.

14. (i) $P(\text{no crash}) = 1 - 10^{-7}$.
(ii) $P(A \mid E_1) + P(A \mid E_2) = 0.95 + 1 = 1.95$.
(iii) (a) $P(A) = 1 - 5 \times 10^{-9}$.

OR

(iii) (b) $P(E_2 \mid A) = \frac{1-10^{-7}}{1-5\times 10^{-9}}$.

15. (i) $P(\text{at least one selected}) = 1 - \frac{4}{5} \times \frac{2}{3} \times \frac{3}{4} = \frac{3}{5}$.
(ii) $P\left(J \mid \overline{R}\right) = \frac{1}{3}$.
(iii) $P(\text{exactly one selected}) = \frac{13}{30}$.

OR

(iii) $P(\text{exactly two selected}) = \frac{3}{20}$.

16. (i) $P(E_1) = 0.7$, $P(E_2) = 0.3$.
(ii) $P(A \mid E_1) = 0.8$, $P(A \mid E_2) = 0.4$.
(iii) $P(A) = 0.7 \times 0.8 + 0.3 \times 0.4 = 0.68$.

OR

(iii) $P(E_1 \mid A) = \frac{0.56}{0.68} = \frac{14}{17}$.

17. (i) $P(\text{all miss}) = \frac{1}{60}$.
(ii) $P(\text{all hit}) = \frac{2}{5}$.
(iii) (a) $P(\text{only one hits}) = \frac{3}{20}$.

OR

(iii) (b) $P(\text{exactly two hit}) = \frac{13}{30}$.