

Bond Valuation

Part 1: Theoretical Concepts (From Scratch)

1. Introduction: What is a Bond?

Concept: Imagine you want to start a business, but you need money. You ask a friend, "Can I borrow Rs. 1,000?" Your friend says, "Okay, but I want Rs. 100 every year as 'rent' for my money, and after 5 years, you must give me back my Rs. 1,000."

In the financial world:

- You are the **Issuer** (Borrower).
- The Friend is the **Investor** (Lender).
- The piece of paper proving this loan is the **Bond**.

2. Key Terms (The Vocabulary)

Before calculating, students must identify the variables.

1. **Face Value (Par Value):** The amount written on the bond certificate. This is the principal amount that is usually repaid at the end.
 - *Example:* Rs. 1,000.
2. **Coupon Rate:** The interest rate the company *promises* to pay. It is always calculated on the **Face Value**.
 - *Example:* 10% coupon on a Rs. 1,000 bond = Rs. 100 interest per year.
3. **Maturity Period:** The lifespan of the bond. When this time ends, the loan is over.
4. **Redemption Value:** The amount the company pays the investor at the end of the maturity period.
 - *Note:* Usually, it is equal to Face Value (at Par). Sometimes, to attract investors, companies pay back *more* than the face value (at Premium).
5. **Current Price (Market Value):** The price at which the bond is currently being bought or sold in the market.
6. **Yield / Effective Rate / Required Rate of Return:** This is the most critical concept. This is the interest rate the **investor wants** or could get elsewhere (like in a bank deposit or another bond).
 - *Crucial Logic:* If the bond pays 8% (Coupon) but the investor wants 10% (Yield), the investor will only buy the bond if the price is lowered (Discount).

Part 2: The First Principle of Valuation

"The value of any asset today is the sum of the Present Values of all future cash flows it will generate."

The Logic (Time Value of Money)

Ask the students: "Would you rather have Rs. 1,000 today or Rs. 1,000 five years from now?"

- They will say "Today."
- **Why?** Because you can invest Rs. 1,000 today and earn interest. Therefore, money in the future is worth **less** than money today.

The Bond Valuation Method: Present Value Approach

To find the fair price of a bond, we must "discount" (bring back) the future money to today's value.

A bond gives the investor two things:

1. **Series of Interest Payments (Coupons):** This happens every year (Annuity).

2. **Repayment of Principal (Redemption):** This happens once at the end (Lump Sum).

The Formula (First Principle):

$$Value = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^n} + \frac{RV}{(1+r)^n}$$

- C = Coupon Amount (Interest)
- r = Required Rate of Return (Yield)
- n = Number of years
- RV = Redemption Value

Note for Class 12: We simplify the summation part using the Annuity Factor.

Simplified Formula:

$$Value = (\text{Coupon Amount} \times PVIFA) + (\text{Redemption Value} \times PVIF)$$

- **PVIFA:** Present Value Interest Factor of Annuity (The multiplier for the series of interest payments).
 - *Calculation:* $\frac{1-(1+r)^{-n}}{r}$
- **PVIF:** Present Value Interest Factor (The multiplier for the final lump sum payment).
 - *Calculation:* $(1+r)^{-n}$

Part 3: Solving the Questions

Here are the step-by-step solutions to the problems you provided, using the principles taught above.

Question 1

"A 2000, 8% bond is redeemable at the end of 10 years at 105. Find the purchase price to yield 10% effective rate. [Given $(1.1)^{-10} = 0.3855$

Step 1: Identify the Variables

- **Face Value (FV):** 2000
- **Coupon Rate:** 8%
- **Annual Coupon (C):** 8% of 2000 = 160
- **Years (n):** 10
- **Required Yield (r):** 10% (or 0.10)
- **Redemption Value (RV):** "Redeemable at 105".
 - *Teacher Note:* In bond markets, a quote of "105" means 105% of the face value.
 - $RV = 2000 \times 105\% = 2000 \times 1.05 = 2100$.

Step 2: Calculate Present Value Factors

- **PVIF (for the lump sum):** Given as $(1.1)^{-10} = 0.3855$
- **PVIFA (for the coupons):** Since we aren't given a table, we use the formula:

$$PVIFA = \frac{1 - PVIF}{r}$$

$$PVIFA = \frac{1 - 0.3855}{0.10}$$

$$PVIFA = \frac{0.6145}{0.10} = 6.145$$

Step 3: Apply the Valuation Formula

$$\text{Price} = (\text{Coupon} \times \text{PVIFA}) + (\text{Redemption Value} \times \text{PVIF})$$

$$\text{Price} = (160 \times 6.145) + (2100 \times 0.3855)$$

Step 4: Calculate

- PV of Coupons = $160 \times 6.145 = 983.2$
- PV of Redemption = $2100 \times 0.3855 = 809.55$
- Total Price = $983.2 + 809.55 = 1792.75$

Answer: The purchase price should be **Rs. 1,792.75**.

Question 2

"Mrs. Jain is considering to buy a Rs. 1,000 par value bond bearing a coupon rate of 11% that matures after 5 years. She wants a minimum rate of return of 15%. The bond is currently sold at Rs. 870. Should she buy the bond? Justify your answer. $[(1.15)^{-5} = 0.4971]$ "

Step 1: Understand the Goal

We need to calculate the **Intrinsic Value** of the bond (what it is worth to *Mrs. Jain*).

- If Calculated Value > Market Price (870) → **Buy** (It's a bargain).
- If Calculated Value < Market Price (870) → **Don't Buy** (It's overpriced).

Step 2: Identify Variables

- **Face Value:** 1,000
- **Coupon (C):** 11% of 1,000 = 110
- **Years (n):** 5
- **Required Yield (r):** 15% (0.15)
- **Redemption Value (RV):** 1,000 (Unless stated otherwise, we assume redemption is at Par).

Step 3: Calculate Factors

- **PVIF:** Given as $(1.15)^{-5} = 0.4971$
- **PVIFA:**

$$PVIFA = \frac{1 - 0.4971}{0.15}$$

$$PVIFA = \frac{0.5029}{0.15} = 3.3526 \text{ (Rounded to 3.353)}$$

Step 4: Calculate Value

$$\text{Value} = (\text{Coupon} \times \text{PVIFA}) + (\text{Redemption Value} \times \text{PVIF})$$

$$\text{Value} = (110 \times 3.3526) + (1000 \times 0.4971)$$

- PV of Coupons = 368.79
- PV of Redemption = 497.10
- **Total Value** = 865.89

Step 5: Conclusion and Justification

- **Value to Mrs. Jain:** Rs. 865.89
- **Current Market Price:** Rs. 870.00

Answer: No, Mrs. Jain **should not buy** the bond.

Justification: The fair value of the bond based on her expected return of 15% is Rs. 865.89. Since the bond is selling

for Rs. 870 in the market, it is **overpriced** relative to her requirements. Buying it would result in a return slightly lower than her 15% target.

EXAMPLES

Category 1: The Relationship between Coupon vs. Yield (The Seesaw Effect)

Concept:

- If Coupon Rate > Required Yield → Bond sells at **Premium** (Price > Face Value).
- If Coupon Rate < Required Yield → Bond sells at **Discount** (Price < Face Value).
- If Coupon Rate = Required Yield → Bond sells at **Par** (Price = Face Value).

Question 1:

A company issues a bond with a Face Value of Rs. 1,000 and a Coupon Rate of 10% payable annually. The bond matures in 5 years and is redeemable at par. Calculate the value of the bond in the following three scenarios:

1. **Case A:** The investor expects a return (Yield) of 8%.
2. **Case B:** The investor expects a return (Yield) of 10%.
3. **Case C:** The investor expects a return (Yield) of 12%.

Given Factors:

- **At 8% (5 years):** PVIF = 0.6806, PVIFA = 3.9927
- **At 10% (5 years):** PVIF = 0.6209, PVIFA = 3.7908
- **At 12% (5 years):** PVIF = 0.5674, PVIFA = 3.6048

Solution for Discussion:

- **Case A (8%):** $(100 \times 3.9927) + (1000 \times 0.6806) = 399.27 + 680.60 = \text{Rs. 1,079.87}$ (Premium)
- **Case B (10%):** $(100 \times 3.7908) + (1000 \times 0.6209) = 379.08 + 620.90 = \text{Rs. 1,000.00}$ (Par)
- **Case C (12%):** $(100 \times 3.6048) + (1000 \times 0.5674) = 360.48 + 567.40 = \text{Rs. 927.88}$ (Discount)

Category 2: Redemption at Premium

Concept: Sometimes companies promise to pay back *more* than the face value at the end to attract investors. This changes the "Lump Sum" part of the formula.

Question 2:

Mr. Sharma wants to invest in a bond issued by Alpha Ltd.

- **Face Value:** Rs. 5,000
- **Coupon Rate:** 12% (Annual)
- **Maturity:** 6 Years
- **Redemption Terms:** The bond will be redeemed at a **premium of 5%**.
- **Mr. Sharma's Required Return:** 14%

Find the maximum price Mr. Sharma should pay.

Given: $(1.14)^{-6} = 0.4556$

Solution:

1. **Calculate Annual Interest:** 12% of 5000 = 600.

2. **Calculate Redemption Value:** $5000 + 5\% = 5,250$.
3. **Calculate PVIFA:** $\frac{1-0.4556}{0.14} = 3.888$.
4. **Formula:** $(600 \times 3.888) + (5,250 \times 0.4556)$
 - PV of Interest = 2,332.8
 - PV of Redemption = 2,391.9
 - **Value = Rs. 4,724.7**

Category 3: Semi-Annual Interest (The Twist)

Concept: Most real-world bonds pay interest every 6 months.

Adjustments needed:

1. **Coupon:** Divide by 2.
2. **Yield (r):** Divide by 2.
3. **Time (n):** Multiply by 2.

Question 3:

A Rs. 1,000 bond pays interest at 10% **semi-annually**. The bond has 4 years remaining until maturity. The market's required rate of return for similar bonds is 12% per annum. Calculate the intrinsic value of the bond.

Given Factors (Note: Use 6% for 8 periods):

$$(1.06)^{-8} = 0.6274$$

Solution:

1. **Adjust Variables:**
 - Annual Coupon = 100 → **Semi-annual Coupon = 50**
 - Annual Yield = 12% → **Semi-annual Yield (r) = 6%**
 - Years = 4 → **Periods (n) = 8**
2. **Calculate PVIFA:** $\frac{1-0.6274}{0.06} = 6.21$.
3. **Calculation:**
 - Value = $(50 \times 6.21) + (1000 \times 0.6274)$
 - Value = 310.5 + 627.4 = **Rs. 937.9**

Category 4: Zero Coupon Bonds (Deep Discount Bonds)

Concept: These bonds pay NO interest during the life. They are sold at a huge discount and paid back at par. The "Annuity" part of the formula becomes zero.

Question 4:

A "Deep Discount Bond" has a face value of Rs. 20,000 and matures in 15 years. No interest is paid during the tenure. If the investor expects a return of 11%, what is the value of this bond today?

Given: $(1.11)^{-15} = 0.209$

Solution:

1. **Formula:** Since there is no coupon, we only calculate the PV of the Redemption Value.

$$\text{Value} = \text{RedemptionValue} \times \text{PVIF}$$

2. **Calculation:**

$$\text{Value} = 20,000 \times 0.209$$

Category 5: Investment Decision (Buy or Sell?)

Concept: Comparing intrinsic value vs. market price.

Question 5:

Global Tech Ltd. has issued bonds with a face value of Rs. 100, carrying an interest rate of 9%. The bond matures in 5 years at Par.

The current market price of the bond is **Rs. 82**.

If your required rate of return is 13%, is this bond a good buy?

Given: $(1.13)^{-5} = 0.5428$

Solution:

1. Find the Intrinsic Value:

- $C = 9, r = 13\%, n = 5, RV = 100.$
- $PVIFA = \frac{1-0.5428}{0.13} = 3.517.$
- Value = $(9 \times 3.517) + (100 \times 0.5428)$.
- Value = $31.65 + 54.28 = \text{Rs. 85.93}$.

2. Decision:

- Value to you: **Rs. 85.93**
- Market Price: **Rs. 82.00**
- **Conclusion: YES, Buy it.** The bond is "Undervalued" (Cheaper than it should be).

Here is a complete worksheet designed for Grade 12 students. It progresses from identifying terms to complex calculations and decision-making.

WORKSHEET: FUNDAMENTALS OF BOND VALUATION

SECTION A: PRACTICE QUESTIONS

Q1. (Concept & Identification)

A company issues a bond with a Face Value of Rs. 1,000. It pays an interest of Rs. 120 every year. The bond will be repaid after 8 years.

(a) What is the **Coupon Rate**?

Q2. (Basic Valuation - Discount)

Calculate the value of a bond with a face value of Rs. 1,000, a coupon rate of 9%, and a maturity of 5 years. The required rate of return (Yield) by investors is 12%.

[Given for 12%, 5 years: PVIF = 0.5674; PVIFA = 3.6048]

Q3. (Redemption at Premium)

Consider a bond with a face value of Rs. 500. The coupon rate is 8% payable annually. The bond matures in 4 years and is **redeemable at a premium of 10%**. Find the value of the bond if the market yield is 10%.

[Given for 10%, 4 years: PVIF = 0.6830; PVIFA = 3.1699]

Q4. (Semi-Annual Interest Application)

Mr. X is analyzing a bond with a face value of Rs. 1,000 that pays a coupon rate of 10% **payable semi-annually** (twice a year). The bond has 3 years remaining until maturity. The market's required effective rate for similar bonds is 12% p.a. Calculate the fair price of the bond.

[Hint: Adjust coupon, rate, and time for semi-annual periods.]

[Given for 6%, 6 periods: $PVIF = 0.7050$; $PVIFA = 4.9173$]

Q5. (Investment Decision - Buy or Sell)

A bond of Rs. 2,000 face value bears a coupon rate of 14% and matures after 4 years at par. The bond is currently trading in the stock market at **Rs. 2,150**. The investor expects a minimum return of 12%.

(a) Calculate the Intrinsic Value of the bond.

(b) Should the investor buy the bond?

[Given for 12%, 4 years: $PVIF = 0.6355$; $PVIFA = 3.0373$]

Q6. (Zero Coupon Bond)

Beta Ltd. issues a "Deep Discount Bond" (Zero Coupon Bond) with a face value of Rs. 25,000 redeemable after 10 years. The investor expects a return of 9%. What is the maximum price the investor should pay for this bond today?

[Given for 9%, 10 years: $PVIF = 0.4224$]

Q7. (Critical Thinking)

Without calculating, determine if the Bond Price will be **Higher**, **Lower**, or **Equal** to the Face Value in the following scenarios:

- (a) Coupon Rate (10%) < Required Yield (12%)
- (b) Coupon Rate (10%) > Required Yield (8%)
- (c) Coupon Rate (10%) = Required Yield (10%)

SECTION B: SOLUTIONS & TEACHER'S NOTES

Solution 1

- **Concept:** Understanding basic terms.
- (a) **Coupon Rate:** $\frac{\text{Annual Interest}}{\text{Face Value}} \times 100 = \frac{120}{1000} \times 100 = 12\%$

Solution 2

- **Given:** Face Value (FV) = 1000, Coupon (C) = 90 (9% of 1000), Yield (r) = 12%, n = 5.
- **Formula:** $V = (C \times PVIFA) + (FV \times PVIF)$
- **Calculation:**

$$V = (90 \times 3.6048) + (1000 \times 0.5674)$$

$$V = 324.43 + 567.40$$

- **Answer: Rs. 891.83**
- **Observation:** Since Yield (12%) > Coupon (9%), the bond is valued at a discount (below 1000).

Solution 3

- **Given:** FV = 500, Coupon (C) = 40 (8% of 500), Yield (r) = 10%, n = 4.
- **Redemption Value (RV):** $500 + 10\% \text{ Premium} = 500 + 50 = 550$.
- **Formula:** $V = (C \times PVIFA) + (RV \times PVIF)$
- **Calculation:**

$$V = (40 \times 3.1699) + (550 \times 0.6830)$$

$$V = 126.80 + 375.65$$

- **Answer: Rs. 502.45**

Solution 4

- **Given:** FV = 1000, Annual Coupon = 10%, Annual Yield = 12%, Years = 3.
- **Adjustments for Semi-Annual:**
 - **C (Semi-annual Coupon):** $100/2 = \text{Rs. 50}$
 - **r (Semi-annual Yield):** $12\%/2 = 6\%$
 - **n (Total Periods):** $3 \times 2 = 6 \text{ periods}$
- **Calculation:**

$$V = (50 \times 4.9173) + (1000 \times 0.7050)$$

$$V = 245.87 + 705.00$$

- **Answer: Rs. 950.87**

Solution 5

- **Given:** FV = 2000, Coupon (C) = 280 (14% of 2000), Yield (r) = 12%, n = 4. Market Price = 2150.
- **Step A (Valuation):**

$$V = (280 \times 3.0373) + (2000 \times 0.6355)$$

$$V = 850.44 + 1271.00$$

Intrinsic Value = Rs. 2,121.44

- **Step B (Decision):**
 - Intrinsic Value (Rs. 2,121.44) < Market Price (Rs. 2,150).
 - The bond is **Overpriced**.
- **Answer:** No, the investor should **NOT** buy the bond.

Solution 6

- **Given:** FV = 25,000, Yield (r) = 9%, n = 10.
- **Note:** Zero Coupon bonds have NO annual interest payments ($C = 0$).
- **Formula:** $V = FV \times PVIF$
- **Calculation:**

$$V = 25,000 \times 0.4224$$

- **Answer: Rs. 10,560**

Solution 7

- (a) **Lower** (Discount)
- (b) **Higher** (Premium)
- (c) **Equal** (Par)