

# Bond Valuation

## Part 1: Theoretical Concepts (From Scratch)

### 1. Introduction: What is a Bond?

**Concept:** Imagine you want to start a business, but you need money. You ask a friend, "Can I borrow Rs. 1,000?" Your friend says, "Okay, but I want Rs. 100 every year as 'rent' for my money, and after 5 years, you must give me back my Rs. 1,000."

In the financial world:

- You are the **Issuer** (Borrower).
- The Friend is the **Investor** (Lender).
- The piece of paper proving this loan is the **Bond**.

### 2. Key Terms (The Vocabulary)

Before calculating, students must identify the variables.

1. **Face Value (Par Value):** The amount written on the bond certificate. This is the principal amount that is usually repaid at the end.
    - *Example:* Rs. 1,000.
  2. **Coupon Rate:** The interest rate the company *promises* to pay. It is always calculated on the **Face Value**.
    - *Example:* 10% coupon on a Rs. 1,000 bond = Rs. 100 interest per year.
  3. **Maturity Period:** The lifespan of the bond. When this time ends, the loan is over.
  4. **Redemption Value:** The amount the company pays the investor at the end of the maturity period.
    - *Note:* Usually, it is equal to Face Value (at Par). Sometimes, to attract investors, companies pay back *more* than the face value (at Premium).
  5. **Current Price (Market Value):** The price at which the bond is currently being bought or sold in the market.
  6. **Yield / Effective Rate / Required Rate of Return:** This is the most critical concept. This is the interest rate the **investor wants** or could get elsewhere (like in a bank deposit or another bond).
    - *Crucial Logic:* If the bond pays 8% (Coupon) but the investor wants 10% (Yield), the investor will only buy the bond if the price is lowered (Discount).
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## Part 2: The First Principle of Valuation

"The value of any asset today is the sum of the Present Values of all future cash flows it will generate."

### The Logic (Time Value of Money)

Ask the students: "Would you rather have Rs. 1,000 today or Rs. 1,000 five years from now?"

- They will say "Today."
- **Why?** Because you can invest Rs. 1,000 today and earn interest. Therefore, money in the future is worth **less** than money today.

### The Bond Valuation Method: Present Value Approach

To find the fair price of a bond, we must "discount" (bring back) the future money to today's value.

A bond gives the investor two things:

1. **Series of Interest Payments (Coupons):** This happens every year (Annuity).

2. **Repayment of Principal (Redemption):** This happens once at the end (Lump Sum).

**The Formula (First Principle):**

$$Value = \frac{C}{(1+r)^1} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^n} + \frac{RV}{(1+r)^n}$$

- $C$  = Coupon Amount (Interest)
- $r$  = Required Rate of Return (Yield)
- $n$  = Number of years
- $RV$  = Redemption Value

*Note for Class 12: We simplify the summation part using the Annuity Factor.*

**Simplified Formula:**

$$Value = (\text{Coupon Amount} \times \text{PVIFA}) + (\text{Redemption Value} \times \text{PVIF})$$

- **PVIFA:** Present Value Interest Factor of Annuity (The multiplier for the series of interest payments).
    - *Calculation:*  $\frac{1-(1+r)^{-n}}{r}$
  - **PVIF:** Present Value Interest Factor (The multiplier for the final lump sum payment).
    - *Calculation:*  $(1+r)^{-n}$
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## Part 3: Solving the Questions

Here are the step-by-step solutions to the problems you provided, using the principles taught above.

### Question 1

**"A 2000, 8% bond is redeemable at the end of 10 years at 105. Find the purchase price to yield 10% effective rate. [Given  $(1.1)^{-10} = 0.3855$ ]"**

#### Step 1: Identify the Variables

- **Face Value (FV):** 2000
- **Coupon Rate:** 8%
- **Annual Coupon (C):** 8% of 2000 = 160
- **Years (n):** 10
- **Required Yield (r):** 10% (or 0.10)
- **Redemption Value (RV):** "Redeemable at 105".
  - *Teacher Note:* In bond markets, a quote of "105" means 105% of the face value.
  - $RV = 2000 \times 105\% = 2000 \times 1.05 = 2100$ .

#### Step 2: Calculate Present Value Factors

- **PVIF (for the lump sum):** Given as  $(1.1)^{-10} = 0.3855$
- **PVIFA (for the coupons):** Since we aren't given a table, we use the formula:

$$PVIFA = \frac{1 - PVIF}{r}$$

$$PVIFA = \frac{1 - 0.3855}{0.10}$$

$$PVIFA = \frac{0.6145}{0.10} = 6.145$$

#### Step 3: Apply the Valuation Formula

$$\text{Price} = (\text{Coupon} \times \text{PVIFA}) + (\text{Redemption Value} \times \text{PVIF})$$

$$\text{Price} = (160 \times 6.145) + (2100 \times 0.3855)$$

#### Step 4: Calculate

- PV of Coupons =  $160 \times 6.145 = 983.2$
- PV of Redemption =  $2100 \times 0.3855 = 809.55$
- Total Price =  $983.2 + 809.55 = 1792.75$

**Answer:** The purchase price should be **Rs. 1,792.75**.

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## Question 2

"Mrs. Jain is considering to buy a Rs. 1,000 par value bond bearing a coupon rate of 11% that matures after 5 years. She wants a minimum rate of return of 15%. The bond is currently sold at Rs. 870. Should she buy the bond? Justify your answer. [ $(1.15)^{-5} = 0.4971$ ]"

#### Step 1: Understand the Goal

We need to calculate the **Intrinsic Value** of the bond (what it is worth *to Mrs. Jain*).

- If Calculated Value > Market Price (870) → **Buy** (It's a bargain).
- If Calculated Value < Market Price (870) → **Don't Buy** (It's overpriced).

#### Step 2: Identify Variables

- **Face Value:** 1,000
- **Coupon (C):** 11% of 1,000 = 110
- **Years (n):** 5
- **Required Yield (r):** 15% (0.15)
- **Redemption Value (RV):** 1,000 (Unless stated otherwise, we assume redemption is at Par).

#### Step 3: Calculate Factors

- **PVIF:** Given as  $(1.15)^{-5} = 0.4971$
- **PVIFA:**

$$PVIFA = \frac{1 - 0.4971}{0.15}$$

$$PVIFA = \frac{0.5029}{0.15} = 3.3526 \text{ (Rounded to 3.353)}$$

#### Step 4: Calculate Value

$$\text{Value} = (\text{Coupon} \times \text{PVIFA}) + (\text{Redemption Value} \times \text{PVIF})$$

$$\text{Value} = (110 \times 3.3526) + (1000 \times 0.4971)$$

- PV of Coupons = 368.79
- PV of Redemption = 497.10
- **Total Value** = 865.89

#### Step 5: Conclusion and Justification

- **Value to Mrs. Jain:** Rs. 865.89
- **Current Market Price:** Rs. 870.00

**Answer:** No, Mrs. Jain **should not buy** the bond.

**Justification:** The fair value of the bond based on her expected return of 15% is Rs. 865.89. Since the bond is selling

for Rs. 870 in the market, it is **overpriced** relative to her requirements. Buying it would result in a return slightly lower than her 15% target.

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## EXAMPLES

### Category 1: The Relationship between Coupon vs. Yield (The Seesaw Effect)

#### Concept:

- If Coupon Rate > Required Yield → Bond sells at **Premium** (Price > Face Value).
- If Coupon Rate < Required Yield → Bond sells at **Discount** (Price < Face Value).
- If Coupon Rate = Required Yield → Bond sells at **Par** (Price = Face Value).

#### Question 1:

A company issues a bond with a Face Value of Rs. 1,000 and a Coupon Rate of 10% payable annually. The bond matures in 5 years and is redeemable at par. Calculate the value of the bond in the following three scenarios:

1. **Case A:** The investor expects a return (Yield) of 8%.
2. **Case B:** The investor expects a return (Yield) of 10%.
3. **Case C:** The investor expects a return (Yield) of 12%.

#### Given Factors:

- **At 8% (5 years):** PVIF = 0.6806, PVIFA = 3.9927
- **At 10% (5 years):** PVIF = 0.6209, PVIFA = 3.7908
- **At 12% (5 years):** PVIF = 0.5674, PVIFA = 3.6048

#### Solution for Discussion:

- **Case A (8%):**  $(100 \times 3.9927) + (1000 \times 0.6806) = 399.27 + 680.60 = \text{Rs. } 1,079.87$  (Premium)
  - **Case B (10%):**  $(100 \times 3.7908) + (1000 \times 0.6209) = 379.08 + 620.90 = \text{Rs. } 1,000.00$  (Par)
  - **Case C (12%):**  $(100 \times 3.6048) + (1000 \times 0.5674) = 360.48 + 567.40 = \text{Rs. } 927.88$  (Discount)
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### Category 2: Redemption at Premium

**Concept:** Sometimes companies promise to pay back *more* than the face value at the end to attract investors. This changes the "Lump Sum" part of the formula.

#### Question 2:

Mr. Sharma wants to invest in a bond issued by Alpha Ltd.

- **Face Value:** Rs. 5,000
- **Coupon Rate:** 12% (Annual)
- **Maturity:** 6 Years
- **Redemption Terms:** The bond will be redeemed at a **premium of 5%.**
- **Mr. Sharma's Required Return:** 14%

Find the maximum price Mr. Sharma should pay.

**Given:**  $(1.14)^{-6} = 0.4556$

#### Solution:

1. **Calculate Annual Interest:** 12% of 5000 = 600.

2. **Calculate Redemption Value:**  $5000 + 5\% = 5,250$ .

3. **Calculate PVIFA:**  $\frac{1-0.4556}{0.14} = 3.888$ .

4. **Formula:**  $(600 \times 3.888) + (5,250 \times 0.4556)$

- PV of Interest = 2,332.8
  - PV of Redemption = 2,391.9
  - **Value = Rs. 4,724.7**
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### Category 3: Semi-Annual Interest (The Twist)

**Concept:** Most real-world bonds pay interest every 6 months.

**Adjustments needed:**

1. **Coupon:** Divide by 2.
2. **Yield (r):** Divide by 2.
3. **Time (n):** Multiply by 2.

**Question 3:**

A Rs. 1,000 bond pays interest at 10% **semi-annually**. The bond has 4 years remaining until maturity. The market's required rate of return for similar bonds is 12% per annum. Calculate the intrinsic value of the bond.

**Given Factors (Note: Use 6% for 8 periods):**

$$(1.06)^{-8} = 0.6274$$

**Solution:**

1. **Adjust Variables:**

- Annual Coupon = 100 → **Semi-annual Coupon = 50**
- Annual Yield = 12% → **Semi-annual Yield (r) = 6%**
- Years = 4 → **Periods (n) = 8**

2. **Calculate PVIFA:**  $\frac{1-0.6274}{0.06} = 6.21$ .

3. **Calculation:**

- Value =  $(50 \times 6.21) + (1000 \times 0.6274)$
  - Value =  $310.5 + 627.4 = \text{Rs. } 937.9$
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### Category 4: Zero Coupon Bonds (Deep Discount Bonds)

**Concept:** These bonds pay NO interest during the life. They are sold at a huge discount and paid back at par. The "Annuity" part of the formula becomes zero.

**Question 4:**

A "Deep Discount Bond" has a face value of Rs. 20,000 and matures in 15 years. No interest is paid during the tenure. If the investor expects a return of 11%, what is the value of this bond today?

**Given:**  $(1.11)^{-15} = 0.209$

**Solution:**

1. **Formula:** Since there is no coupon, we only calculate the PV of the Redemption Value.

$$\text{Value} = \text{Redemption Value} \times \text{PVIF}$$

2. **Calculation:**

$$\text{Value} = 20,000 \times 0.209$$

$$Value = \text{Rs. } 4,180$$

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## Category 5: Investment Decision (Buy or Sell?)

**Concept:** Comparing intrinsic value vs. market price.

### Question 5:

Global Tech Ltd. has issued bonds with a face value of Rs. 100, carrying an interest rate of 9%. The bond matures in 5 years at Par.

The current market price of the bond is **Rs. 82**.

If your required rate of return is 13%, is this bond a good buy?

**Given:**  $(1.13)^{-5} = 0.5428$

**Solution:**

#### 1. Find the Intrinsic Value:

- $C = 9, r = 13\%, n = 5, RV = 100$ .
- $PVIFA = \frac{1-0.5428}{0.13} = 3.517$ .
- $\text{Value} = (9 \times 3.517) + (100 \times 0.5428)$ .
- $\text{Value} = 31.65 + 54.28 = \text{Rs. } 85.93$ .

#### 2. Decision:

- Value to you: **Rs. 85.93**
  - Market Price: **Rs. 82.00**
  - **Conclusion: YES, Buy it.** The bond is "Undervalued" (Cheaper than it should be).
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Here is a complete worksheet designed for Grade 12 students. It progresses from identifying terms to complex calculations and decision-making.

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## WORKSHEET: FUNDAMENTALS OF BOND VALUATION

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### SECTION A: PRACTICE QUESTIONS

#### Q1. (Concept & Identification)

A company issues a bond with a Face Value of Rs. 1,000. It pays an interest of Rs. 120 every year. The bond will be repaid after 8 years.

(a) What is the **Coupon Rate**?

#### Q2. (Basic Valuation - Discount)

Calculate the value of a bond with a face value of Rs. 1,000, a coupon rate of 9%, and a maturity of 5 years. The required rate of return (Yield) by investors is 12%.

[Given for 12%, 5 years:  $PVIF = 0.5674$ ;  $PVIFA = 3.6048$ ]

#### Q3. (Redemption at Premium)

Consider a bond with a face value of Rs. 500. The coupon rate is 8% payable annually. The bond matures in 4 years and is **redeemable at a premium of 10%**. Find the value of the bond if the market yield is 10%.

[Given for 10%, 4 years:  $PVIF = 0.6830$ ;  $PVIFA = 3.1699$ ]

#### Q4. (Semi-Annual Interest Application)

Mr. X is analyzing a bond with a face value of Rs. 1,000 that pays a coupon rate of 10% **payable semi-annually** (twice a year). The bond has 3 years remaining until maturity. The market's required effective rate for similar bonds is 12% p.a. Calculate the fair price of the bond.

[Hint: Adjust coupon, rate, and time for semi-annual periods.]

[Given for 6%, 6 periods:  $PVIF = 0.7050$ ;  $PVIFA = 4.9173$ ]

#### Q5. (Investment Decision - Buy or Sell)

A bond of Rs. 2,000 face value bears a coupon rate of 14% and matures after 4 years at par. The bond is currently trading in the stock market at **Rs. 2,150**. The investor expects a minimum return of 12%.

(a) Calculate the Intrinsic Value of the bond.

(b) Should the investor buy the bond?

[Given for 12%, 4 years:  $PVIF = 0.6355$ ;  $PVIFA = 3.0373$ ]

#### Q6. (Zero Coupon Bond)

Beta Ltd. issues a "Deep Discount Bond" (Zero Coupon Bond) with a face value of Rs. 25,000 redeemable after 10 years. The investor expects a return of 9%. What is the maximum price the investor should pay for this bond today?

[Given for 9%, 10 years:  $PVIF = 0.4224$ ]

#### Q7. (Critical Thinking)

Without calculating, determine if the Bond Price will be **Higher**, **Lower**, or **Equal** to the Face Value in the following scenarios:

(a) Coupon Rate (10%) < Required Yield (12%)

(b) Coupon Rate (10%) > Required Yield (8%)

(c) Coupon Rate (10%) = Required Yield (10%)

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## SECTION B: SOLUTIONS & TEACHER'S NOTES

### Solution 1

- **Concept:** Understanding basic terms.
- (a) **Coupon Rate:**  $\frac{\text{Annual Interest}}{\text{Face Value}} \times 100 = \frac{120}{1000} \times 100 = 12\%$

### Solution 2

- **Given:** Face Value (FV) = 1000, Coupon (C) = 90 (9% of 1000), Yield (r) = 12%, n = 5.
- **Formula:**  $V = (C \times PVIFA) + (FV \times PVIF)$
- **Calculation:**

$$V = (90 \times 3.6048) + (1000 \times 0.5674)$$

$$V = 324.43 + 567.40$$

- **Answer: Rs. 891.83**
- **Observation:** Since Yield (12%) > Coupon (9%), the bond is valued at a discount (below 1000).

### Solution 3

- **Given:** FV = 500, Coupon (C) = 40 (8% of 500), Yield (r) = 10%, n = 4.
- **Redemption Value (RV):**  $500 + 10\% \text{ Premium} = 500 + 50 = 550$ .
- **Formula:**  $V = (C \times PVIFA) + (RV \times PVIF)$
- **Calculation:**

$$V = (40 \times 3.1699) + (550 \times 0.6830)$$

$$V = 126.80 + 375.65$$

- Answer: Rs. 502.45

#### Solution 4

- **Given:** FV = 1000, Annual Coupon = 10%, Annual Yield = 12%, Years = 3.
- **Adjustments for Semi-Annual:**
  - **C (Semi-annual Coupon):**  $100/2 = \text{Rs. } 50$
  - **r (Semi-annual Yield):**  $12\%/2 = 6\%$
  - **n (Total Periods):**  $3 \times 2 = 6 \text{ periods}$
- **Calculation:**

$$V = (50 \times 4.9173) + (1000 \times 0.7050)$$

$$V = 245.87 + 705.00$$

- Answer: Rs. 950.87

#### Solution 5

- **Given:** FV = 2000, Coupon (C) = 280 (14% of 2000), Yield (r) = 12%, n = 4. Market Price = 2150.
- **Step A (Valuation):**

$$V = (280 \times 3.0373) + (2000 \times 0.6355)$$

$$V = 850.44 + 1271.00$$

**Intrinsic Value = Rs. 2,121.44**

- **Step B (Decision):**
  - Intrinsic Value (Rs. 2,121.44) < Market Price (Rs. 2,150).
  - The bond is **Overpriced**.
- **Answer:** No, the investor should **NOT** buy the bond.

#### Solution 6

- **Given:** FV = 25,000, Yield (r) = 9%, n = 10.
- **Note:** Zero Coupon bonds have NO annual interest payments ( $C = 0$ ).
- **Formula:**  $V = FV \times PVIF$
- **Calculation:**

$$V = 25,000 \times 0.4224$$

- Answer: Rs. 10,560

#### Solution 7

- (a) **Lower** (Discount)
- (b) **Higher** (Premium)
- (c) **Equal** (Par)