Domain & Range of Function

▼ Find the domain of given functions

Level 1: Simple Denominators

1. $f(x) = \frac{10}{x-5}$
2. $f(x) = \frac{3x+7}{2x-8}$
3. $f(x) = \frac{x^2-4}{x}$

Level 2: Simple Square Roots

1. $g(x) = \sqrt{x-1}$
2. $g(x) = \sqrt{2x+12}$
3. $g(x) = \sqrt{10-x}$

Level 3: Quadratic Denominators

1. $h(x) = \frac{x+1}{x^2-16}$
2. $h(x) = \frac{1}{x^2-5x}$
3. $h(x) = \frac{2x}{x^2+4x+3}$

Level 4: Combining Square Roots and Denominators

1. $k(x) = \frac{1}{\sqrt{x-6}}$
2. $k(x) = \frac{\sqrt{x+2}}{x-3}$
3. $k(x) = \frac{x-5}{\sqrt{20-4x}}$

Level 5: Quadratic Expressions Inside Square Roots

1. $p(x) = \sqrt{x^2-9}$
2. $p(x) = \sqrt{x^2-7x+10}$

3. $p(x) = \sqrt{25 - x^2}$

Level 6: Domains Involving Modulus (Notion/LaTeX Format)

Problems

1. $f(x) = \frac{5}{|x|-3}$
2. $g(x) = \frac{x-1}{|2x+8|}$
3. $h(x) = \sqrt{|x-1|}$
4. $k(x) = \sqrt{|x| - 7}$
5. $m(x) = \frac{1}{\sqrt{9-|x|}}$

Answer Key

1. **Answer:** $(-\infty, 5) \cup (5, \infty)$
2. **Answer:** $(-\infty, 4) \cup (4, \infty)$
3. **Answer:** $(-\infty, 0) \cup (0, \infty)$
4. **Answer:** $[1, \infty)$
5. **Answer:** $[-6, \infty)$
6. **Answer:** $(-\infty, 10]$
7. **Answer:** $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
8. **Answer:** $(-\infty, 0) \cup (0, 5) \cup (5, \infty)$
9. **Answer:** $(-\infty, -3) \cup (-3, -1) \cup (-1, \infty)$
10. **Answer:** $(6, \infty)$
11. **Answer:** $[-2, 3) \cup (3, \infty)$
12. **Answer:** $(-\infty, 5)$
13. **Answer:** $(-\infty, -3] \cup [3, \infty)$
14. **Answer:** $(-\infty, 2] \cup [5, \infty)$
15. **Answer:** $[-5, 5]$

Answer Key (Modulus Set)

1. **Answer:** $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$
2. **Answer:** $(-\infty, -4) \cup (-4, \infty)$
3. **Answer:** $(-\infty, \infty)$
4. **Answer:** $(-\infty, -7] \cup [7, \infty)$
5. **Answer:** $(-9, 9)$

▼ Part-2

Problem Set

1. $f(x) = x^2 - 4$
2. $g(x) = \sqrt{x-3} + 1$
3. $h(x) = \frac{5}{x+2}$
4. $k(x) = 8 - x^2$
5. $m(x) = |x+2| - 3$
6. $p(x) = \frac{2x+1}{x-4}$
7. $q(x) = \sqrt{25 - x^2}$
8. $s(x) = 4 - \sqrt{x+5}$
9. $t(x) = \frac{6}{x^2+3}$

Answer Key

1. **Function:** $$ f(x) = x^2 - 4 $$
    - **Domain:** $(-\infty, \infty)$
        - *Reasoning:* It's a polynomial with no denominators or roots, so there are no restrictions on the input *x*.
    - **Range:** $[-4, \infty)$
        - *Reasoning:* The term x^2 has a minimum value of 0. Therefore, the minimum value of the function is $0 - 4 = -4$.
2. **Function:** $$ g(x) = \sqrt{x-3} + 1 $$
    - **Domain:** $[3, \infty)$

        - *Reasoning:* The expression inside the square root must be non-negative: $x-3 \ge 0 \implies x \ge 3$.
    - **Range:** $[1, \infty)$
        - *Reasoning:* The output of $\sqrt{x-3}$ is always ≥ 0. Therefore, the minimum output of the entire function is $0 + 1 = 1$.
3. **Function:** $$ h(x) = \frac{5}{x+2} $$
    - **Domain:** $(-\infty, -2) \cup (-2, \infty)$
        - *Reasoning:* The denominator cannot be zero: $x+2 \neq 0 \implies x \neq -2$.
    - **Range:** $(-\infty, 0) \cup (0, \infty)$
        - *Reasoning:* A fraction can only be zero if its numerator is zero. Since the numerator is 5, this fraction can never equal 0, but it can produce any other non-zero number.
4. **Function:** $$ k(x) = 8 - x^2 $$
    - **Domain:** $(-\infty, \infty)$
        - *Reasoning:* It's a polynomial with no restrictions.
    - **Range:** $(-\infty, 8]$
        - *Reasoning:* The term x^2 is always ≥ 0. So, $-x^2$ is always ≤ 0. The maximum value of $-x^2$ is 0, making the function's maximum value $8 - 0 = 8$.
5. **Function:** $$ m(x) = |x+2| - 3 $$
    - **Domain:** $(-\infty, \infty)$
        - *Reasoning:* The absolute value function accepts any real number input.
    - **Range:** $[-3, \infty)$
        - *Reasoning:* The term $|x+2|$ has a minimum output of 0. Therefore, the minimum output of the entire function is $0 - 3 = -3$.
6. **Function:** $$ p(x) = \frac{2x+1}{x-4} $$
    - **Domain:** $(-\infty, 4) \cup (4, \infty)$

    - *Reasoning:* The denominator cannot be zero: $x-4 \neq 0 \implies x \neq 4$.
  - **Range:** $(-\infty, 2) \cup (2, \infty)$
    - *Reasoning:* Using the inverse method, solving $y = \frac{2x+1}{x-4}$ for *x* gives $x = \frac{4y+1}{y-2}$. The denominator of this inverse shows that $y \neq 2$.

7. **Function:** $$ q(x) = \sqrt{25 - x^2} $$
   - **Domain:** $[-5, 5]$
     - *Reasoning:* We need $25 - x^2 \ge 0 \implies 25 \ge x^2 \implies -5 \le x \le 5$.
   - **Range:** $$
     - *Reasoning:* The function represents the top half of a circle with radius 5. The minimum output is 0 (when $x=\pm 5$). The maximum output is 5 (when $x=0$).
8. **Function:** $$ s(x) = 4 - \sqrt{x+5} $$

- **Domain:** $[-5, \infty)$
  - *Reasoning:* We need $x+5 \ge 0 \implies x \ge -5$.
- **Range:** $(-\infty, 4]$
  - *Reasoning:* The term $\sqrt{x+5}$ is always ≥ 0. So, $-\sqrt{x+5}$ is always ≤ 0. The maximum value of this part is 0. Thus, the function's maximum value is $4 - 0 = 4$.

1. **Function:** $$ t(x) = \frac{6}{x^2+3} $$
   - **Domain:** $(-\infty, \infty)$
     - *Reasoning:* The denominator x^2+3 is always positive (minimum value is 3), so it can never be zero.
   - **Range:** $(0, 2]$
     - *Reasoning:* The denominator x^2+3 has a minimum value of 3 (when $x=0$). This creates the fraction's maximum value: $6/3 = 2$. As *x* approaches $\pm\infty$, the denominator becomes infinitely large, making the fraction approach, but never reach,

0.pm\infty$, the denominator becomes infinitely large, making the fraction approach, but never reach, 0.