PT3 Mathematics Worksheet

1. Find the domain of each of the following functions:
   (i) $f(x) = \frac{x}{x^2+3x+2}$
   (ii) $f(x) = [x] + x$

2. Find the range of the following functions:
   (i) $\frac{|x-4|}{x-4}$
   (ii) $\sqrt{16 - x^2}$

3. Find the domain of each of the following functions:
   (i) $f(x) = \frac{1}{\sqrt{1-\cos x}}$
   (ii) $f(x) = \frac{1}{\sqrt{x+|x|}}$
   (iii) $f(x) = x|x|$
   (iv) $f(x) = \frac{x^3-x+3}{x^2-1}$
   (v) $f(x) = \frac{3x}{2x-8}$

4. Find the range of the following functions:
   (i) $f(x) = \frac{3}{2-x^2}$
   (ii) $f(x) = 1 - |x - 2|$
   (iii) $f(x) = |x - 3|$
   (iv) $f(x) = 1 + 3\cos 2x$

5. Solve the following system of inequalities:
   $\frac{x}{2x+1} \geq \frac{1}{4}, \frac{6x}{4x-1} < \frac{1}{2}$

6. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

7. The longest side of a triangle is twice the shortest side and the third side is 2cm longer than the shortest side. If the perimeter of the triangle is more than 166 cm then find the minimum length of the shortest side.

8. Solve the following system of inequalities:
   $\frac{2x+1}{7x-1} > 5, \frac{x+7}{x-8} > 2$

9. If the letters of the word RACHIT are arranged in all possible ways as listed in dictionary. Then what is the rank of the word RACHIT?

10. Find the number of different words that can be formed from the letters of the word 'TRIANGLE' so that no vowels are together.

11. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated.

12. If $^nC_{r-1} = 36, {}^nC_r = 84$ and $^nC_{r+1} = 126$, then find $^rC_2$.

13. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if the team has:
    (i) no girls
    (ii) at least one boy and one girl
    (iii) at least three girls

14. Evaluate: $(1 + i)^6 + (1 - i)^3$

15. If $(x + iy)^{\frac{1}{3}} = a + ib$, where $x, y, a, b \in R$, show that $\frac{x}{a} - \frac{y}{b} = -2(a^2 + b^2)$.

16. What is the value of $\frac{i^{4n+1}-i^{4n-1}}{2}$?

17. What is the smallest positive integer n, for which $(1+i)^{2n} = (1-i)^{2n}$?

18. What is the conjugate of $\frac{\sqrt{5+12i}+\sqrt{5-12i}}{\sqrt{5+12i}-\sqrt{5-12i}}$?

19. For a positive integer n, find the value of $(1-i)^n \left(1-\frac{1}{i}\right)^n$.

20. Evaluate $\sum_{n=1}^{13} (i^n + i^{n+1})$, where $n \in \mathbb{N}$.

21. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = x + iy$, then find (x, y).

22. If $\frac{(1+i)^2}{2-i} = x + iy$, then find the value of $x + y$.

23. If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then find (a, b).

24. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. Find the common ratio of the G.P.

25. In a G.P. of positive terms, if any term is equal to the sum of the next two terms. Then find the common ratio of the G.P.

26. If the pth and qth terms of a G.P. are q and p respectively, show that its $(p+q)^{th}$ term is $\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$.

27. Show that the tangent of an angle between the lines $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{a} - \frac{y}{b} = 1$ is $\frac{2ab}{a^2-b^2}$.

28. Find the equation of the line which passes through the point $(-4, 3)$ and the portion of the line intercepted between the axes is divided internally in the ratio $5 : 3$ by this point.

29. Find the equations of the lines through the point of intersection of the lines $x - y + 1 = 0$ and $2x - 3y + 5 = 0$ and whose distance from the point $(3, 2)$ is $\frac{7}{5}$.

30. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a^2, p^2, b^2 are in A.P, then show that $a^4 + b^4 = 0$.

31. If the origin is the centroid of a triangle ABC having vertices $A(a, 1, 3)$, $B(-2, b, -5)$ and $C(4, 7, c)$, find the values of a, b, c.

32. Let $A(2, 2, -3)$, $B(5, 6, 9)$ and $C(2, 7, 9)$ be the vertices of a triangle. The internal bisector of the angle A meets BC at the point D. Find the coordinates of D.

33. Find the centroid of a triangle, the mid-point of whose sides are $D(1, 2, -3)$, $E(3, 0, 1)$ and $F(-1, 1, -4)$.

34. There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test. Determine the mean and standard deviation of the marks (x is a positive integer).

Marks	0	1	2	3	4	5
Frequency	$x - 2$	x	x^2	$(x+1)^2$	$2x$	$x + 1$

35. Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

36. If A and B are mutually exclusive events, $P(A) = 0.35$ and $P(B) = 0.45$, find:
(a) $P(A')$
(b) $P(B')$
(c) $P(A \cup B)$
(d) $P(A \cap B)$
(e) $P(A \cap B')$
(f) $P(A' \cap B')$