the XY-plane. Obviously PL || RN || QM and feet of these perpendiculars lie in a XY-plane. The points L, M and N will lie on a line which is the intersection of the plane containing PL, RN and QM with the XY-plane. Through the point R draw a line ST parallel to the line LM. Line ST will intersect the line LP externally at the point S and the line MQ at T, as shown in Fig 12.5.

Also note that quadrilaterals LNRS and NMTR are parallelograms.

The triangles PSR and QTR are similar. Therefore,

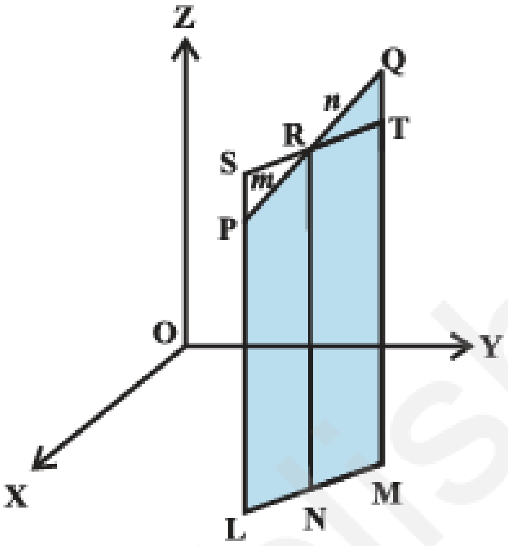

Fig 12.5

$$\frac{m}{n} = \frac{\text{PR}}{\text{QR}} = \frac{\text{SP}}{\text{QT}} = \frac{\text{SL} - \text{PL}}{\text{QM} - \text{TM}} = \frac{\text{NR} - \text{PL}}{\text{QM} - \text{NR}} = \frac{z - z_1}{z_2 - z}$$

This implies $$z = \frac{mz_2 + nz_1}{m+n}$$

Similarly, by drawing perpendiculars to the XZ and YZ-planes, we get

$$y = \frac{my_2 + ny_1}{m+n} \text{ and } x = \frac{mx_2 + nx_1}{m+n}$$

Hence, the coordinates of the point R which divides the line segment joining two points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio $m : n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n}\right)$$

If the point R divides PQ externally in the ratio $m : n$, then its coordinates are obtained by replacing n by $-n$ so that coordinates of point R will be

$$\left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n}\right)$$

Case 1 Coordinates of the mid-point: In case R is the mid-point of PQ, then

$$m : n = 1 : 1 \text{ so that } x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2} \text{ and } z = \frac{z_1 + z_2}{2}.$$

These are the coordinates of the mid point of the segment joining P (x_1, y_1, z_1) and Q (x_2, y_2, z_2).

EXERCISE 12.3

1. Find the coordinates of the point which divides the line segment joining the points (– 2, 3, 5) and (1, – 4, 6) in the ratio (i) 2 : 3 internally, (ii) 2 : 3 externally.
2. Given that P (3, 2, – 4), Q (5, 4, – 6) and R (9, 8, –10) are collinear. Find the ratio in which Q divides PR.
3. Find the ratio in which the YZ-plane divides the line segment formed by joining the points (–2, 4, 7) and (3, –5, 8).
4. Using section formula, show that the points A (2, –3, 4), B (–1, 2, 1) and $C\left(0,\frac{1}{3},2\right)$ are collinear.
5. Find the coordinates of the points which trisect the line segment joining the points P (4, 2, – 6) and Q (10, –16, 6).